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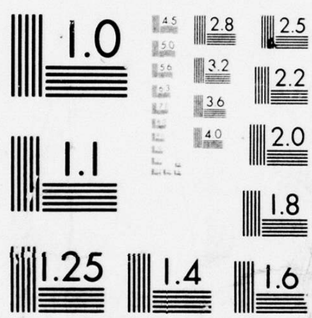

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This study examined the assumption that the underlying failure distribution for inertial measurement units is exponential. Failure data from three inertial measurement units, the KT-73 unit from the A-7D/E and AC-130H, the FLIP unit from the C-5A, and the LN-15 unit from the B-52G/H, were analyzed by use of the SIMFIT computer program. This program is a curve fitting technique employed for fitting a sample of data to twelve theoretical distributions. The research hypothesis was to show that the "exponential assumption" of failures for the above units was inappropriate by finding a failure distribution which would better model the failure data. The results indicated that the gamma was, in fact, the best model of the distributions tested and several other models, such as the Weibull and lognormal, were generally found to be better than the exponential model. Other areas of reliability discussed include renewal, data aggregation, and infant mortality.

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AN ANALYSIS OF THE EXPONENTIAL FUNCTION AS THE UNDERLYING DISTRIBUTION FOR DESCRIBING FAILURES IN INERTIAL MEASUREMENT UNITS

A Thesis

Presented to the Faculty of the School of Systems and Logistics

of the Air Force Institute of Technology

Air University

In Partial Fulfillment of the Requirements for the

Degree of Master of Science in Logistics Management

By

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June 1977

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This thesis, written by

Captain Lowell R. Crowe

and

Captain Levi D. Lowman, Jr.

has been accepted by the undersigned on behalf of the faculty of the School of Systems and Logistics in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE IN LOGISTICS MANAGEMENT

DATE: 15 June 1977

COMMITTEE CHAIRMAN

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CHAPTER I

INTRODUCTION

An ever-growing dependence upon sophisticated technology for success in military endeavors has intensified the importance of reliability. At the same time, continuing increases in modern weapon system complexity have made the achievement of this reliability extremely difficult. Intuitively, equipment with a high degree of reliability implies that the equipment will operate longer without failure and will have a greater likelihood of being ready for use when needed. More technically, reliability is defined in terms of probability. Specifically, reliability is the probability that a system, component, item, unit, or piece of equipment will adequately perform its intended function for a required interval of time under expected operating conditions (10:205).

Terminology

Because of the many technical terms and mathematical concepts utilized in discussing the complex nature of reliability theory, a list of definitions is inserted early in the thesis for reader familiarization. This initial introduction of definitions will be reinforced by expanded explanations of many of the terms as they occur throughout

the research effort. This reinforcement technique is used to carefully guide the reader through the technical discussions required in the thesis. Terminology given will be for the purpose of clarity and consistency within this thesis.

Aggregated data. Data points which have been grouped together from two or more samples or populations so that the initial source is no longer identifiable.

Availability. A measure of the degree to which a unit is operable at the start of a mission at some random point in time.

Burn-in. The early operation of a unit to stabilize its operational characteristics.

Catastrophic failure. A failure which occurs suddenly without warning and which results in a complete failure of a unit.

Chi-square test (χ^2). A Goodness-Of-Fit (G-O-F) test for determining whether or not a reliability function based on observed data can be described by a specific theoretical function.

Component/Item/Unit. Interchangeable terms for any part, subassembly, line replaceable unit, system, or piece of equipment whose failure and replacement must be considered in reliability analysis.

Component population. The total number of units in a given situation from which samples may be drawn.

Constant maintenance policy. A maintenance policy which does not change over the period of time that is being considered.

Convolution. The process of randomly selecting values from different samples and then summing these values to create a new random variable.

Debugging. A reliability procedure used to detect and eliminate early failures and thus stabilize the operation of a unit.

Exponential distribution. The exponential variation of the probability of occurrence of a variable with respect to a change in time.

Failure. The inability of a unit to perform adequately; i.e., the opposite of "operating normally." For purposes of this research, a unit has only two states of nature; either the unit is operational, or the unit has failed.

Failure rate. The number of failures of a unit per interval measure of time.

Hazard rate. The instantaneous failure rate of the components in a system under consideration; or, the limit of the failure rate as the interval length approaches zero.

Infant mortality. The premature catastrophic failure in the early operating life of a unit which occurs at a rate substantially greater than would normally be expected.

Mean Time Between Failure (MTBF). For a given time interval, the total functioning life of a population of a unit divided by the total number of failures within the population during the measurement interval. _This MTBF definition is applicable only to the exponential distribution_. For other statistical distributions, the MTBF calculation must be computed on an individual basis.

Planned life. The expected operational life of a unit at the time of acquisition.

Random failure. A failure whose occurrence is predictable only in a probabilistic or statistical sense.

Simulation. A set of test conditions designed to duplicate operating and usage environment and conditions as closely as possible.

Steady state. The normal expected operating state for a given unit. For the exponential case, the steady state is the condition of a constant failure (hazard) rate.

Wearout. An attrition process which results in an increase of the failure rate with increasing age (32:10).

Statement of the Problem

The Air Force implicitly assumes that unit failures follow an exponential distribution in establishing the procedures for determining support requirements based on the number of failures. The use of the exponential distribution implies a constant failure rate with respect to time for any component population. A constant failure rate, which is also referred to as a constant hazard rate, infers that all failures occur stochastically (randomly). Therefore, the Air Force's use of exponential distribution with its associated constant hazard rate implies that a unit's failure is as apt to occur after the first hour of operation as it is to occur after a thousand hours of operation (26). The "exponential assumption" ignores, or assumes to be insignificant, the realities of infant mortality and/or wearout (aging) as considerations for a unit's failure. Instead, emphasis is shifted to the computational simplicity of the use of the exponential distribution by designing failure data collection systems based on the parameter,

Mean Time Between Failures (MTBF), of the exponential distribution.

Under the exponential assumption of a constant failure rate, only the total operating time and the total number of failures are needed to mathematically compute a point estimate of the MTBF (35). Many current data collection systems for assessing reliability are designed only to record these failure data. But this level of detail is only sufficient for the determination of the MTBF, the one parameter that describes the exponential distribution. Therefore, the assumption of the exponential function as describing the underlying failure model based on this combined or aggregated failure data is made out of sheer necessity (28:3).

The use of the exponential MTBF formula suggests a unit's immediate entry into a steady state condition. Steady state is a condition in which failures occur at a constant rate; i.e., the condition of a constant hazard rate. Thus, an assumption of the exponential function as a model for describing a unit's failure process implies that the unit always operates in a steady state condition. But, in fact, computer simulation models indicate that the time to reach steady state may range up to sixteen years under certain maintenance policies (8:72-7; 11:53).

The problem is that the current Air Force assumption of a constant failure rate may be erroneous until a steady

state is actually, if ever, attained. If so, the errors introduced into logistics planning result in misstated support requirements and thus erroneous cost estimates. These requirements include how many spare parts are to be purchased, how many support manhours are to be scheduled, what support facilities are to be made available, and even the number of units that are required to be purchased to support a given mission (8).

Justification for the Research

Today, in the Air Force, the total cost of support for a given system is at least equal to the initial acquisition cost; therefore, a need exists to constantly question current logistical practices, including those involving reliability. The Air Force needs to continually search for more accurate, economical procedures and methods to apply to the attainment of a specified mission. With sky-rocketing inflation and increasing attention by Congress on Air Force and Department of Defense expenditures, challenges to out-dated ways of performing specific functions must be made to provide a strong defense posture at the lowest total cost.

The maintenance of a strong defense by the United States requires the acquisition of the best, most reliable weapon systems that are available. But if the capability to obtain these weapon systems is lessened by the cost of

the weapon systems' design and associated support requirements, ways and means must be found and used to decrease these costs while still providing effective weapon systems. Primary justification of this thesis rests on the premise that the exponential assumption of component/system failures may be <u>inappropriately applied</u> in many situations and, thus influence the cost of the design, acquisition, and support of modern weapon systems.

This thesis challenges the common practice of assuming that unit/equipment failures are generated from a stochastic process whose underlying distribution is exponential. If the exponential assumption is indeed inappropriate, then the related costs of making such an assumption may be substantial.

Management needs current, accurate information about unit/equipment failure distributions to determine necessary resource requirements. These requirements include planning the number and time of reordering spares, anticipating maintenance and other support manpower, and other considerations during the planned life of the components/systems. In this age of spiraling costs for these requirements and of increased demands on modern weapon systems, challenges must be made to established practices and paradigms in a continual search for the best way to cope with increased requirements and decreased resources.

Military Assumptions

Air Force Regulation (AFR) 80-5, Reliability and Maintainability Programs for Systems, Subsystems, Equipment, and Munitions, states that reliability influences logistical requirements and, therefore, is an essential part of an effective weapon system (37). Current trends toward the acquisition of weapon systems based on life cycle cost criteria, including logistical support, dictate that reliability be considered throughout the system's total life. In conjunction with these reliability considerations throughout the Air Force, AFR 80-5 states that adequate data reporting systems will be provided to aid management in assessing weapon system reliability.

These data collection systems are normally based on the criteria as set forth by the military standards (MIL-STD) specified by the DOD procuring agency at the time of the systems' acquisition. MIL-STD-785, Requirements for Military Programs (for Systems and Equipment/Development and Production), is the primary reliability publication used in the preparation of specifications and contractural documents by the DOD for contractor reliability programs (36:1). This publication permits the contractor to assume that equipment failures follow the exponential distribution wherever this military standard is referenced in the contract (36:8).

MIL-STD-781B, Reliability Tests: Exponential Distribution, also implies that failures are exponentially distributed; in fact, the entire military standard is fundamentally based on this assumption. This document outlines reliability qualifications, reliability production acceptance, and longevity tests (35:1).

MIL-STD-756, Reliability Prediction, facilitates the use of the exponential distribution in military reliability prediction (34:2, 5). The DOD generally assumes " . . . in their contracting and specification reliability standard that the single most important parameter is the MTBF [8:15]." The parameter MTBF is defined in MIL-STD-721B, Definitions of Effectiveness Terms for Reliability, Maintainability, Human Factors, and Safety, as " . . . the total functioning life of a population of an item divided by the total number of failures within the population during the measurement interval . . . [32:5]."[1]

This standard definition of MTBF supports the use of the exponential assumption of failures in the development of data collection systems and serves to encourage the aggregation of failure data. Such aggregated data generally consist of only the total operating time and the total number of failures. Although these data are sufficient for

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[1]A listing of other documents of interest used by the Air Force in military reliability prediction and contractual specifications is presented in Appendix A.

evaluating the MTBF parameter of the exponential distribution, they are useless in determining the parameters of two-parameter distributions such as the gamma and Weibull.

Fortunately, a few data collection systems maintained by depot level repair facilities do not aggregate data. These data systems record operating times, the number of failures for a given unit, and other data that are necessary in a failure analysis. When data are collected in this manner, parameters can be calculated for one or two parameter distributions. The one parameter exponential model can then be compared with two parameter models, such as the gamma and Weibull, to determine which is the most appropriate model for a given set of failure data.

Scope of the Research

This thesis was restricted to a literature search and the analysis of failure data obtained from three inertial measurement units (IMU). Failure data were analyzed using the SIMFIT computer program.[2] Although there is a comprehensive set of distributions for the purpose of fitting failure data within SIMFIT, their number is limited. While other distributions which could possibly model the failure data better than the ones contained in SIMFIT, the

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[2] SIMFIT is a computer program which tests data to determine if the data follow a theoretical distribution. An explanation of SIMFIT is contained in Appendix B.

most common and most widely used model, namely the exponential, is contained within SIMFIT. This distribution was of primary interest throughout the research.

An additional limitation may be the SIMFIT computer program's use of the nonparametric Chi-square (χ^2) and the Kolomogorov-Smirnov (K-S) Goodness-Of-Fit (G-O-F) tests. However, these nonparametric curve fitting methods reflect the current state of the art.[3]

Objective of the Research

The objective of this thesis was to investigate and compare the distribution of actual failure rates, computed using operational field data, with theoretical distributions of expected failure rates. Failure data for independently failing units were collected and used to compute parameters for field data to compare with those specified in the contract at the time of unit acquisition. The fact that these units were renewed after each failure had to be considered during the evaluation procedure.

The major research task was to determine if the exponential distribution was the appropriate statistical model for each renewal cycle in the life of the population.

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[3] See Appendix C for a complete discussion of the K-S and χ^2 G-O-F tests.

Research Hypothesis

The distribution of failures for some replaceable/ repairable units follow a statistical distribution which differs from the exponential distribution model.

CHAPTER II

RELIABILITY CONCEPTS AND METHODS

As components/systems have become more complex, the applicable reliability concepts (and models) have become increasingly more sophisticated. This component/system complexity has increased the degree and number of mathematical concepts needed to understand modern reliability theory. These reliability concepts require a basic understanding of probability and statistical theory because both the laws of probability and the techniques of statistics are needed to accurately describe and model failures of given units.[1]

This chapter describes the reliability and statistical concepts that are necessary to understand the research effort. First, the concept of a hazard rate is presented with a discussion of how the gamma and Weibull distributions can be used to model different hazard rates. Next, the exponential distribution is shown to be a special case of the Weibull and gamma models when their parameters are appropriately chosen to represent a constant hazard

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[1]A development of the probability concepts and statistical techniques used in this thesis can be found in most reliability texts such as Shooman (26).

rate. Following the discussion of the exponential model, the concept of renewal is introduced. Finally, attention is directed to the SIMFIT computer program, which was used to evaluate the research hypothesis concerning the inappropriate use of the exponential distribution as a model for failure data.

Hazard Rate

One of the most useful tools in the study of reliability is the concept of a hazard rate, $H(t)$, defined as the conditional probability of failure in the time interval $(t, t + \Delta t)$, given that the unit has survived up to time t. Stated another way, $H(t)$ is the ratio of the expected number of failures in an interval of time to the number of units presently operating during that interval of time (13:83).

The hazard rate is mathematically defined as

$$H(t) \equiv - \lim_{\Delta t \to 0} \frac{n(t) - n(t + \Delta t)}{n(t)\Delta t} ,$$

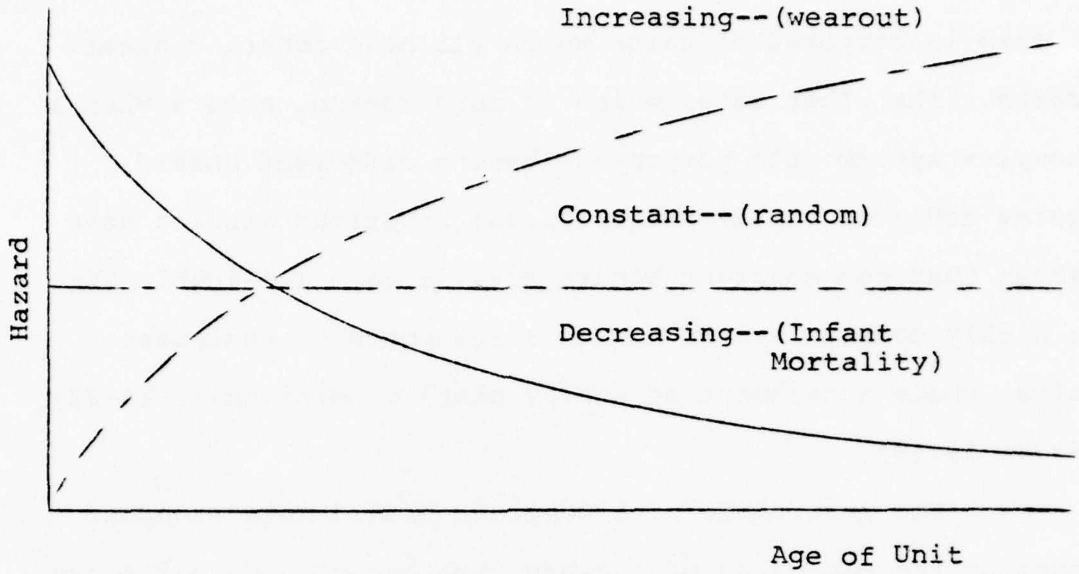
where $n(t) = N \cdot R(t)$, and N represents the number of units surviving at time t, Δt represents an increment of time, and $R(t)$ is the reliability of a unit at the beginning of the interval $(t, t + \Delta t)$. A more common form of the hazard rate used in reliability is

$$H(t) = \frac{f(t)}{R(t)} ,$$

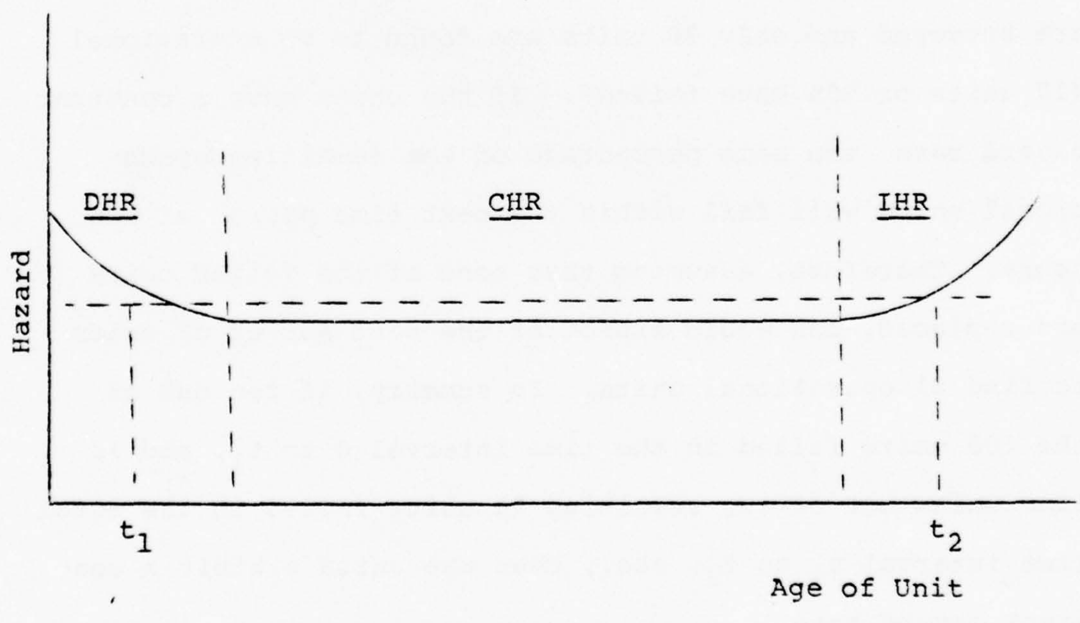
where f(t) is the probability density function of the unit (26:182).

The hazard rate is an instantaneous failure rate of a unit under consideration. If it can be shown that the failure rate for a given unit is independent of time, then the unit is said to have a constant hazard rate. One reason for the use of the hazard rate, particularly the constant hazard rate, is that it allows relatively easy computation and easily interpreted graphs. Figure 2.1a depicts the graphs of a constant hazard rate (CHR), as well as those of an increasing hazard rate (IHR) and a decreasing hazard rate (DHR).

A constant hazard rate implies that the probability of failure in any time period remains constant throughout a given unit's lifetime. For many mechanical units, which generally deteriorate or wear out over time, the hazard rate is not constant, but increasing. On the other hand, many electronic units/systems often exhibit a decreasing hazard rate which is the phenomenon referred to as infant mortality (26:185-9). For a system composed of units which have different individual hazard rates, a system hazard rate will result. The system hazard rate consisting of first a decreasing hazard rate, then a constant hazard rate, and finally an increasing hazard rate has been named the "bath tub" curve (Figure 2.1b) (26:194).



(a) Simple Shapes



(b) Compound Shape

Figure 2.1. Hazard Rates [(a) Simple Shapes, (b) Compound Shape]

Erickson and Hammond have shown that two situations lead to the use of a CHR. The first case occurs when the system is composed of units which all have constant hazard rates. The other case, which is more common, occurs when a complex system with components having different hazard rates achieves steady state (11:24). Various studies have shown that the constant hazard rate is more applicable for a highly complex system or for large pools of equipment after their attainment of steady state conditions (2:18-22; 6:78; 19:49).

For an example of a constant hazard rate, suppose that at the beginning of a given time period, 100 units are operational. After some time period (ten hours), the units are surveyed and only 90 units are found to be operational (10 units or 10% have failed). If the units have a constant hazard rate, the same percentage of the remaining operational units will fail within the next time period of ten hours. Therefore, assuming that none of the failed units are replaced, one would expect at the next survey of units to find 81 operational units. In summary, if ten out of the 100 units failed in the time interval 0 to t_1, and if nine units out of the remaining 90 units failed in the equal time interval t_1 to t_2, etc., then the units exhibit a constant hazard rate.

Weibull Distribution

The fact that the failures of a component population may exhibit properties of the "bath tub" curve or other mixtures of hazard rates has generated a great deal of interest about the Weibull and gamma distributions, within the world of reliability. The Weibull model is not only capable of representing a constant hazard rate but also can be used to model an increasing hazard rate or a type of decreasing hazard rate by the appropriate selection of the model parameters (26:190).

The Weibull distribution does exhibit two drawbacks. First, the model is a two parameter model, which means it is more difficult to estimate the parameters. But with the availability of modern computers and talented programmers, this drawback should be minor. The second drawback is within the model itself. The Weibull model cannot accurately represent the linearly decreasing hazard rate, which is useful in describing early failures. "However, with an appropriate choice of k and m [the model's parameters] one should be able to minimize this effect [26:190]."

The Weibull density function is mathematically defined as

$$f(t) = kt^{m} \exp\left[\frac{-kt^{(m+1)}}{(m+1)}\right]$$

where m and k are the models' parameters.[2]

In the Weibull model, if m = 0, the resulting distribution is the exponential distribution. Therefore, the exponential distribution can be thought of as a special case of the Weibull distribution, or alternatively, the Weibull distribution can be thought of as a more general case of the exponential.

Gamma Distribution

Another failure model which proved to be important during this research was the two parameter form of the gamma distribution. Similar to the Weibull model, the gamma distribution has the ability to model different hazard rates, including the constant hazard rate, for various component populations.

The gamma density function is mathematically defined as

$$f(t) = \left[\beta^{\alpha+1}\,\Gamma(\alpha + 1)\right]^{-1} t^{\alpha} \exp\left[-\frac{t}{\beta}\right]$$

---

[2]For additional development of the Weibull distribution see Appendix D.

where $\alpha > -1$, $\beta > 0$, $0 \leq t \leq \infty$, and $\Gamma(x) = (x - 1)!$, when x is an integer (23:4-37).

If $\alpha = 0$, the exponential distribution is obtained as a special case of the gamma distribution. And, if α is defined as being a positive integer, the resulting distribution is called the Erlang distribution.[3]

Exponential Distribution

Since the exponential distribution is a special case of the Weibull or gamma distributions, one may conjecture that if an underlying distribution has been assumed exponential by an oversimplification of reality, then a more appropriate distribution might be the Weibull or gamma. But the fact is, however, reliability analysis of failures has concentrated on the concept of a constant hazard rate. To assume a component/system's hazard rate is constant implies that the failures of the component/system follow an exponential distribution.

A comprehensive analysis of failure distributions by the RAND Corporation in the early 1950's discovered that the exponential distribution accurately described the failure characteristics for a wide variety of devices (17:209). The list of applications where the exponential distribution

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[3]See Appendix D for further development of the gamma and Erlang distributions and a summarized discussion of other statistical distributions considered important to the analysis of the IMU failure data.

could be used grew rapidly, but there were arguments against the universal use of the exponential distribution.

> The exponential law is reasonable appropriate where chance alone dictates failure occurrence. If it is known, for example, that failure is consistently due to deterioration, wearout, degradation fatigue, or any repetitive mechanism, almost certain nonexponentiality is implied . . . . To be truly random, failure cannot be due to design deficiencies or manufacturing errors. Being assignable as to cause, failure distributions associated with early life of a product usually are not exponential in that they are not random [21:194].

And

> . . . it is to be emphasized that considerable care is required in the selection of the appropriate underlying distribution for reliability testing. The validity of the test results depends to a large degree upon how well the selected probability distribution represents the actual distribution of the time to failure upon which observations are being made . . . . This point is emphasized because the widespread (perhaps indiscriminate) use of the exponential distribution as a model of failure patterns may lead one to believe that failure times in general may be adequately represented by such a distribution . . . . [7:3-6].

The exponential distribution is a consequence of the assumption that the probability of failure in a given time interval is directly proportional to the length of the interval and is independent of the age of the unit. This assumption means that individual failures occur in a random or unpredictable manner and are not caused by design imperfections (infant mortality) or deterioration (wearout) (26:24).

The exponential density function is mathematically defined as

$$f(t) = \lambda \exp[-\lambda t], \ t \geq 0$$

and the reliability function for the distribution is given as

$$R(t) = \exp[-\lambda t],$$

where $1/\lambda = \Theta$ is the Mean Time to Failure (MTTF). The expected value of the exponential distribution, $E(t) = \Theta$, while the variance, $V(t) = \Theta^2$ and the standard deviation is equal to Θ. A very interesting relationship concerning the exponential parameter, MTTF, is that it is equal to the standard deviation. This relationship further facilitates the simplicity in using the exponential distribution.

The MTTF is itself a popular measure of reliability as is the MTBF.

> The MTBF has meaning only when one is discussing a renewal situation, where there is repair or replacement. . . . Unfortunately these two quantities [MTTF & MTBF] are sometimes wrongly thought of as equivalent, probably because for certain simple constant-hazard cases they are equal. In a single-parameter distribution, specification of the MTTF fixes the parameter [26:197].

In multiple parameter distributions, such as the Weibull and gamma, the MTBF places constraints on only one of the model's parameters. Whereas, the MTBF completely constrains the single parameter exponential distribution.

Renewal

The Mean Time Between Failures is so frequently encountered in reliability that an understanding of the concept of renewal or repair of a unit is vital. Physically, renewal implies that when a unit fails, replacement is normally made with an identical operable unit or the failed unit is repaired by maintenance actions which completely restores the unit's operational capabilities. This repair or replacement of failed units is called a renewal process (13:97).

Since renewal is a mathematical, as well as a physical concept, a more quantitative development of the renewal concept is desirable. For example, let t_1 be the time of the first failure of a given unit; at which, the first unit is either replaced by another operable, identical unit or completely repaired. In renewal theory, the downtime for the removal and replacement process and any shelf life in the supply system of a unit is usually assumed to be negligible. If the second unit (or renewed unit) begins operation at time t_1 and eventually fails at t_1', the total operating time is given by $t_1' - t_1 = t_2$ time periods. The procedure continues with unit three (or unit one renewed for the second time) operating t_3 time periods, etc. The system operating time T_n for n - 1 renewals is given by

$$T_n = t_1 + t_2 + t_3 + \cdot \cdot \cdot + t_n,$$

where T_n is the time of the n^{th} failure and t_n is the time from the $(n-1)^{st}$ failure to the n^{th} failure.

Each $f(T_n)$ is now a density function for n renewals computed from the individual density functions $f_n(t_n)$. This procedure is called a convolution. The convolution of a set of individual density functions implies that a random observation is extracted from each of the individual density functions and then summed. These summations are now random observations in their own rights of the convoluted density function T_n (26:350-9).

For example, assume that a population of operable identical units exists in which only one unit is needed to satisfactorily perform a particular mission. If unit one is initially placed in operation at time 0 and fails after 15 hours, the total operating time of unit one is $t_1 = 15$ hours. If unit one is replaced by unit two and subsequently fails after operating 26 hours, the total operating time for unit two is $t_2 = 26$ hours. Unit two is now replaced by unit three and after nine hours, unit three fails with the total operating time for unit three being $t_3 = 9$. If the example is stopped at this point, the system operating time is

$$T_n = t_1 + t_2 + t_3 = (15 + 26 + 9) \text{ hours} = 50 \text{ hours}.$$

Now if one assumes that the population contains m units and that the operating times for t_1, t_2, and t_3 were randomly

drawn from failure cycles 1, 2, and 3 respectively, T_n can be conceptualized as a convolution.

In a renewal environment the exponential distribution model is often used for describing failures of units because the model is characterized by a "complete lack of memory" property,

$$P\{X > r + s \mid X > r\} = P\{x > s\},$$

where r and s are any positive numbers (12:156).

> This means that $P\{x>s\}$ is independent of r. In other words, if a piece of equipment has not failed during r time units its conditional probability of serving r+s or more time units is independent of r and is equal to the probability of serving s or more time units. Stated differently, if time to failure of a piece of equipment follows the exponential distribution, then aging of the equipment is immaterial [12:157].

The lack of memory property of the exponential model greatly encourages the distribution's use in considering renewed units. If a unit is assumed to have a constant hazard rate, then when observed at any time t, such that, $(t > 0)$, the unit will theoretically be as good after renewal as the unit was initially. Therefore, renewal of an exponentially distributed unit implies that it will perform the same after the 1000^{th} renewal as it did initially.

For example, consider the system of 100 operational units explained previously. If the ten units which failed in the first time period 0 to t_1 are completely renewed

instantly, then 100 units are again operational at t_1. This constant hazard system with complete renewal would then predict ten units failing in the equal time period t_1 to t_2. In fact, the lack of memory property and complete renewal allows one to assume that ten units would fail in any equal time period selected.

Armed with such reliability concepts as the hazard rate, renewal, and underlying failure distributions, attention is shifted to the basic [illegible] for performing the research.

Research Plan

Failure data for three Inertial Measurement Units (IMU) were obtained from the G078C Data Collection System. This data collection system is maintained by the Aerospace Guidance and Metrology Center (AGMC) at Newark AFS, Ohio. These raw failure data were used to extract useable failure data, such as operating times and failure cycle numbers for the IMU. After the data extraction process, a graphical curve fitting procedure was needed to aid in the selection of the failure data's underlying failure distribution. Thus, the SIMFIT computer program, a 'state of the art' curve fitting technique, was utilized in the comparison of the failure data with theoretical distributions.

Two very important reasons for establishing an underlying distribution for a given set of failure data are:

1. The implication that future failure patterns will be identical with the failure patterns observed in the past (7:3-6).

2. The ability to use statistical techniques and methods based on known distributions in describing the reliability of a given unit.

The SIMFIT computer program performs a curve fitting evaluation routine by performing two nonparametric goodness-of-fit tests. These nonparametric tests, the Kolmogorov-Smirnov (K-S) and the Chi-square (χ^2) Goodness-Of-Fit (G-O-F) tests, are used due to their distribution-free property.[4] This property and the availability of the K-S and χ^2 test results motivated the use of SIMFIT for data analysis. The results of the K-S and χ^2 tests were used to accept or reject a particular distribution as the underlying distribution for a given set of failure data at the 90% confidence level. The statistical decision rules were as follows:

1. Hypothesis Statement(s)

Null	H_o: X ~	the hypothesized distribution with the desired parameter(s).
Alternate	H_1: X $\not\sim$	the hypothesized distribution with the desired parameter(s).

[4] A basic description of nonparametric statistics is contained in Appendix C.

2. Rejection Decision Criteria using the K-S G-O-F Test Results from SIMFIT:

   If the calculated value of D is greater than the critical value of D, reject the null hypothesis at the specified level of confidence.

3. Rejection Decision Criteria using the χ^2 G-O-F Test Results from SIMFIT:

   If the calculated value of χ^2 is greater than the critical value of χ^2, reject the null hypothesis at the specified level of confidence.

Since the SIMFIT computer program employs both of these G-O-F tests, a decision was necessary in the event that the results of the two tests were not the same. Therefore, in this research, if a distribution passed either the K-S or the χ^2 G-O-F Test (or both), the failure data distribution was accepted as following the theoretical distribution with the distribution parameters calculated by SIMFIT.

In order to determine if the research hypothesis was supported, the SIMFIT results from each particular IMU population were analyzed. The analysis was accomplished by grouping the failure data by failure cycle. The best model for a particular population was considered to be the theoretical distribution that fit the largest percentage of failure cycles. If the best model was not the exponential for any of the three IMU populations tested, the research hypothesis was supported. Conversely, if the best model for each population was the exponential distribution, then the research hypothesis was not supported.

Research Assumptions

The following assumptions were made for the research effort:

1. The sample units are independent of each other; therefore, the failure of one unit has no affect on the failure of any other unit in the sample.

2. The renewal process is a function of the renewed/replaced unit. The effects of any improper maintenance actions were not considered. "Conceptually, maintenance malpractice is a random variable [8:32]."

3. SIMFIT selected the best parameters for a given distribution using the optimal number of cells and cell widths.

4. The data provided by the G078C Data Collection System was not contaminated.

5. The populations under study were subjected to a constant maintenance policy.

In the following chapter, the procedures necessary to accomplish the SIMFIT analysis are described in the sequence in which they were actually performed during the research effort.

CHAPTER III

METHODOLOGY

The objective of the research was to compare actual field data with theoretical distributions to determine if the exponential model was indeed the appropriate failure model. To accomplish this objective, the reliability concepts discussed in Chapter II were employed. In addition, a methodology was developed to collect and analyze the failure data. This chapter explains the procedure which were used to meet the research objective.

The population of units, from which the failure data were collected, is addressed initially, along with the criteria used for sample determination. Next the data collection procedure, as well as, a summary of the G078C Data Collection System from which the failure data was extracted, is presented. Finally the techniques of analysis which were used are discussed.

Population and Sample

The universe of the research effort was defined to be the Inertial Measurement Units repaired by the Aerospace Guidance and Metrology Center (AGMC). From this universe three populations were identified for analysis. The nomenclatures associated with the populations are: the FLIP

unit used on the C-5A aircraft, the LN-15 unit used on the B-52G/H aircraft, and the KT-73 unit used on both the A-7D/E aircraft and the AC-130H aircraft (29:5A). For clarity and consistency the nomenclatures FLIP, KT-73, and LN-15, used by the G078C Data Collection System, were employed throughout this research. All available failure data on these three populations were obtained from the AGMC for analysis.

The FLIP, LN-15, and KT-73 populations appeared to have been subjected to a constant maintenance policy during the data collection period. This policy involved the removal and replacement of any unit at the time of failure. Such maintenance was performed at the flight line level. The failed units were then sent to AGMC, where the failures were verified and, if necessary, the units were repaired. Repair consisted of unsealing the hermetically-sealed unit, removing and replacing (R & R) failed sub-units, resealing the unit, and performing an operational checkout (18). The unit was then returned to the supply channel (See Figure 3.1).

The assumption of a constant maintenance policy was critical to the research because if the maintenance policy changes during the collection of data, the observed distribution may also change. If the maintenance policy had changed, the research assumption of a constant maintenance policy stated in Chapter II would not have been met.

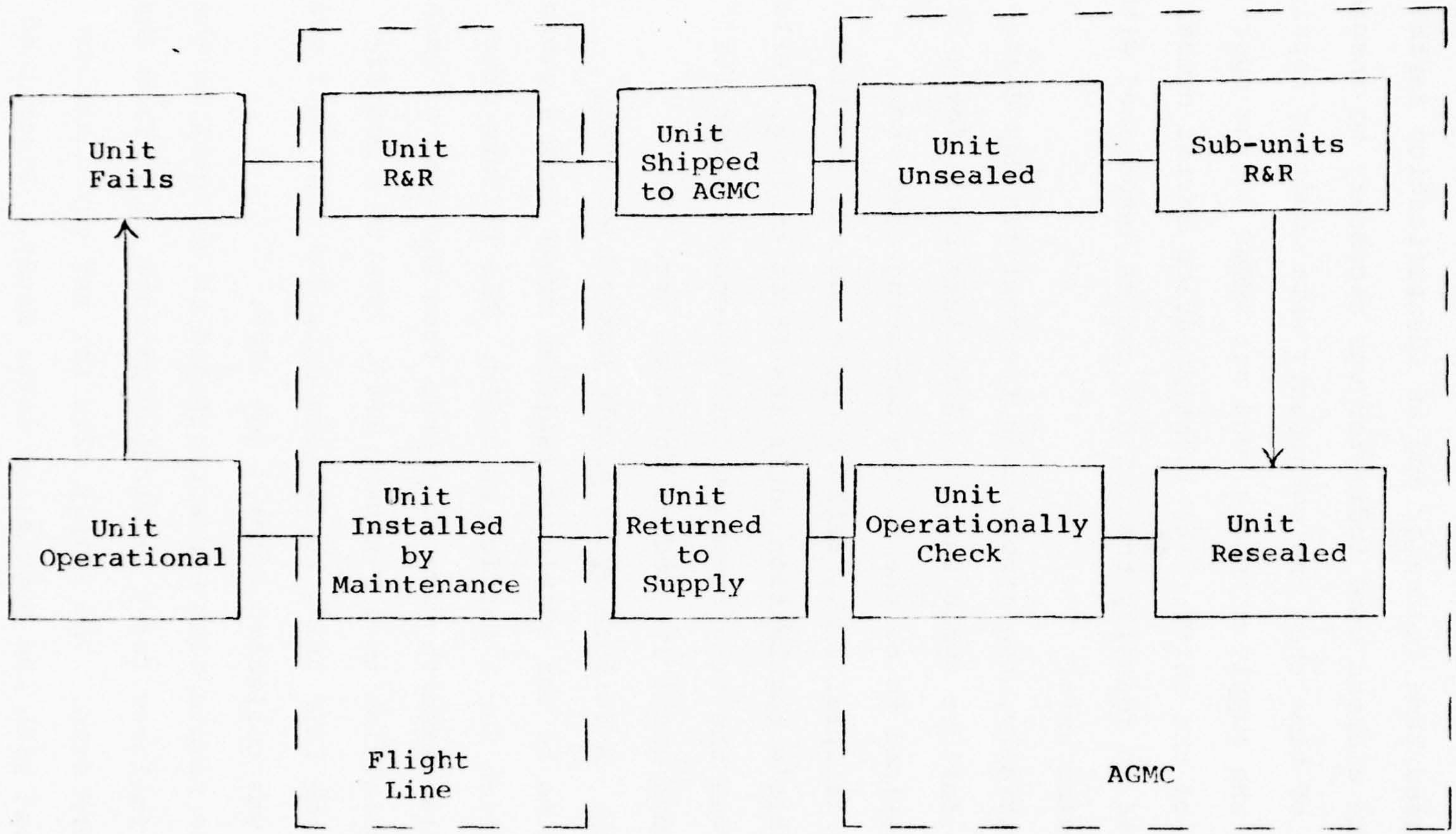
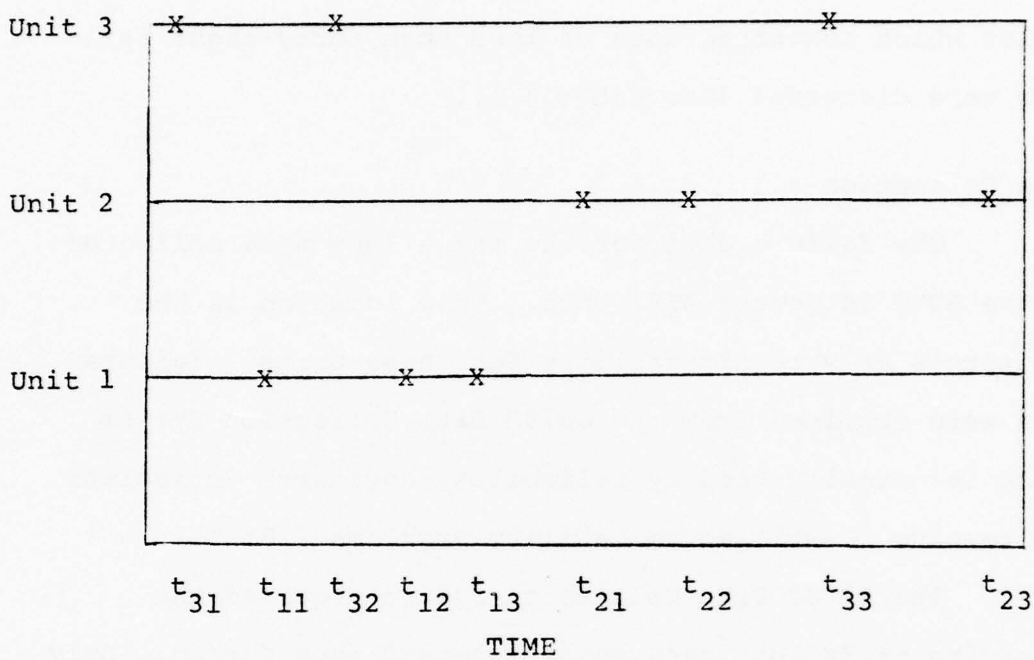


Figure 3.1. Simplified Repair Flow for an IMU

Each individual unit of each population under study had an elapsed time indicator and an identification serial number. The elapsed time indicator was necessary to insure the length of time that the components were awaiting repair, or were in the supply channels, was not added to the operating time of the units. The identification serial number was required to identify the failure cycles associated with any particular unit.

Initially, the sample used for analysis was designated as a failure cycle sample. A failure cycle for a unit was defined to be the unit's operating time from renewal to failure; i.e., failure cycle one was the time from the unit's acquisition until its first failure, failure cycle two was the time from the unit's renewal after its first failure until its second failure, etc. (See Figure 3.2). A failure cycle sample was subjectively determined to be the sample of failures drawn from a given IMU population for a particular cycle. All failure data used in this research were collected from the time of each unit's acquisition to 1 February 1977. Due to a modification of the FLIP unit, no failure data for that unit was used that was collected after 1 May 1976.

The sample size was established to be equal to the number of failures for a particular failure cycle from the AGMC failure data. The sample size for any given failure cycle varied with the population under study. Several of



Where X on the chart represents a failure and renewal.
t_{ij} = j^{th} failure for the i^{th} unit

Failure Cycle 1 Data Sample	Failure Cycle 2 Data Sample	Failure Cycle 3 Data Sample
t_{11} t_{21}	$(t_{12} - t_{11})$	$(t_{13} - t_{12})$
t_{31}	$(t_{22} - t_{21})$	$(t_{23} - t_{22})$
	$(t_{32} - t_{31})$	$(t_{33} - t_{32})$

Figure 3.2. An Illustration of Grouping Failure Data by Failure Cycle

the failure cycles had as few as ten failures. Any result obtained from such a small sample would be questionable; therefore, for the purpose of this research, all failure cycles which contained data on less than forty-eight failures were discarded (See Table 3.1).

Data Collection

The failure data for the three IMUs were collected at the AGMC in Newark AFS, Ohio. This location is the military's only repair facility for these units. Failure data were obtained from the G078C Data Collection System which is normally used by reliability engineers to isolate and resolve identified reliability problems (18:29).

The G078C Data Collection System required the recording of failure data on a five-card type format. Only card type one contained data useful in this research, such as operating hours, number of the failure cycles for each unit, etc.; therefore, only card one data was obtained and analyzed.

The data were sorted by population type and failure cycle. The hours of operation of each unit within each population were extracted from the data cards for every failure cycle. These operating times were then placed into data files by population and failure cycle. These data files were called failure cycle samples and constituted the data base entered into SIMFIT for the analysis. Individual failure cycle operating times are included in Appendix E.

Table 3.1

Summary of Sample Sizes

Failure Cycle	Number of FLIP Failures	Number of KT-73 Failures	Number of LN-15 Failures
1	145	554	316
2	136	451	213
3	127	354	117
4	124	258	49
5	118	185	26
6	110	120	18
7	96	42	7
8	82	23	4
9	62	11	2
10	49	8	0
11	27	1	0
12	11	0	0

Note: All samples with less than forty-eight data points were discarded.

The validity of SIMFIT's test is dependent upon the accuracy of the data collected by AGMC via the G078C Data Collection System. Data validity was a research goal; but, realistically, the possibility existed that the data could have been contaminated for one unit or one cycle. After an extensive search for accurate and useable data, the decision was made to use the G078C data simply because no better, nonaggregated data were available.

Technique of Analysis

Best model criteria. The failure cycle sample operating times were entered into the SIMFIT computer program for each population and compared to the theoretical distributions that were of interest to this research effort; namely, the exponential, Erlang, Weibull, gamma, Pearson XI, lognormal, normal, beta, and negative binomial. The suggested values calculated by the SIMFIT program were used to establish the number of cells and the width of each cell. All distributions were tested at the 90% confidence level because "Current military specifications in contracts dealing with reliability append a 90% confidence factor to the specified reliability measure . . . [4:5]."

One of the outputs from the SIMFIT program was a histogram of the data that were entered. The histograms obtained for the failure cycle samples are contained in Appendix F.

Other SIMFIT outputs included the results of the K-S and χ^2 G-O-F tests. If one of these G-O-F tests "passes" a given theoretical distribution, then the data sample could have been drawn from that distribution. Conversely, if both of these G-O-F tests "fail" a given theoretical distribution, then the data sample could not have been drawn from that distribution. Of course, G-O-F tests are dependent upon the confidence level desired in the pass/fail determination. A 90% confidence level was used in this research. These test results were tabularized in order to determine the best model for each population.

Data aggregation. Another approach to analyzing the failure data of each population was to aggregate all failure cycle samples of a given population into one data file of operating times. In essence, there was one aggregation of failure data for each population. The data aggregation for each population was then tested using SIMFIT to determine any underlying distribution.

Convoluted sampling. An alternate approach to analyzing the appropriateness of the exponential assumption was the method of convolution of data. The convolution approach was used to make the analysis more complete and to possibly add support to the results of the previous analysis. The disadvantage of this approach was that the exponential assumption can only be disproven by this method.

In other words, this was a negative test which could only show that the exponential assumption was inappropriate.

The convolution was accomplished by random sampling with replacement from each failure cycle sample of the specified population. The operating times drawn from each failure cycle were summed and the summation was placed into a data file to be used by SIMFIT. Each of these data files was called a convoluted sample. See Figure 3.3 for a pictorial representation of convolution. To remain consistent with previous analysis of data, all convoluted samples contained more than forty-eight data points.

Reliability comparison. If the underlying distribution for a data sample and/or population can be discovered, then other analysis based on the parametric statistics of that distribution may be performed (22). One such method of analysis was made by comparing the parameters calculated by the SIMFIT program for the actual failure distribution with the parameters specified by the contract at the time of acquisition. Difficulty was encountered in researching the contract reliability specification for the FLIP unit; therefore, no reliability comparison was made between the actual and specified reliability for this unit.

The reliability R(t) and hazard rate H(t) were graphically compared for certain failure cycles. The purpose was to determine if there were any significant

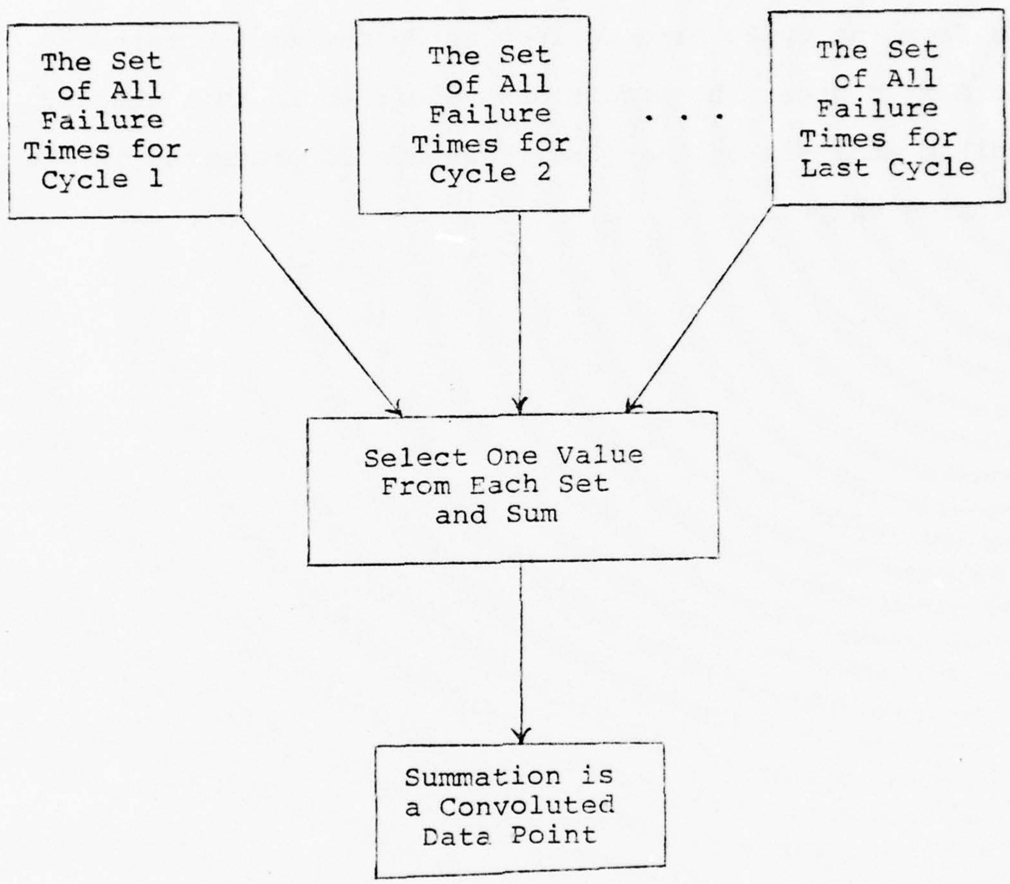


Figure 3.3. A Process of Convolution for IMU Failure Data

differences between the reliability of the actual failure distributions and the contract specified reliability which assumed the exponential distribution.

Once the research methodology was formulated, the data from the G078C Data Collection System were entered into SIMFIT under the conditions specified in this chapter. Specific analysis of the SIMFIT outputs is presented in the following chapter.

CHAPTER IV

ANALYSIS OF RESULTS

This chapter presents an interpretation of the outputs of the SIMFIT computer program using the methodology described in Chapter III. First, the results of the SIMFIT computer program are summarized in tables to facilitate the comparison of the exponential distribution with the other distributions tested via SIMFIT. The purpose was to determine if the exponential distribution was the most appropriate failure model for the populations of Inertial Measurement Units considered. Next, data aggregation test results are presented and explained, followed by a similar summary of the SIMFIT output obtained for the convoluted samples. Finally, a comparison is made of the contract specified reliability and the achieved reliability based on parameters calculated by SIMFIT from the LN-15 and KT-73 failure data.

Certain unexpected results that were encountered during the SIMFIT output analysis prompted research beyond that which was specified in the original methodology. This chapter concludes with an analysis of these anomalies which included a relatively high infant mortality and a shift in failure distributions from cycle to cycle.

Analysis

Best model determination. The best model for each unit was defined to be the distribution which was able to fit the largest number of failure cycle samples for that unit. The results of the SIMFIT data analysis are summarized for the KT-73 unit in Table 4.1.[1] The best model for the KT-73 unit was the gamma distribution; hence, for the KT-73 unit, the research hypothesis is supported. The exponential model did not pass any distribution of failure cycle data.

Table 4.2 summarizes the results of the SIMFIT analysis for the FLIP unit. The FLIP unit failure cycle samples were able to fit the gamma distribution for nine of the ten failure cycles tested. The high percentage of failure cycles that passed the gamma distribution indicated that it also was the best model for describing the pattern of failures for the FLIP unit. Therefore, for the FLIP IMU, the research hypothesis was supported. The exponential distribution, on the other hand, passed five of the ten distributions for the failure cycles tested; and thus, it was not considered as good a model as the gamma distribution.

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[1] Parameters of all distributions passing G-O-F tests via SIMFIT are presented in Tables 4.1, 4.2, and 4.3.

Table 4.1

Summary of the SIMFIT Results for the KT-73 Unit

Cycle	Exponential	Erlang	Weibull	Gamma	Pearson XI	Lognormal	Beta	Normal	Negative Binomial
1	Failed	Passed $K=2$	Passed $\alpha=38409$ $\beta=1.549$	Passed $K=2$ $A=0.0$	Failed	Failed	Passed $A=.48$ $\Gamma=3.476$	Failed	Failed
2	Failed	Failed	Failed	Failed	Failed	Failed	Failed	Failed	Failed
3	Failed	Failed	Failed	Failed	Failed	Failed	Failed	Failed	Failed
4	Failed	Failed	Failed	Failed	Failed	Failed	Failed	Failed	Failed
5	Failed	Failed	Failed	Passed $K=1$ $A=0$	Failed	Passed AHAT=4.064 BSQ=3.646	Failed	Failed	Failed
6	Failed	Failed	Failed	Passed $K=1$ $A=0$	Failed	Passed AHAT=73.36 BSQ=3.455	Failed	Failed	Failed

Note: This table includes the parameters calculated by SIMFIT for the distributions which passed a G-O-F test.

Table 4.2

Summary of the SIMFIT Results for the FLIP Unit

Cycle	Expo-nential	Erlang	Weibull	Gamma	Pearson XI	Lognormal	Beta	Normal	Negative Binomial
1	Failed	Failed	Passed α= 6799.66 β= 1.2826	Passed K=1 A=0	Failed	Failed	Failed	Failed	Failed
2	Failed	Failed	Failed	Failed	Passed	Failed	Failed	Failed	Failed
3	Passed λ= .00333	Passed K=1	Passed α= 229.35 β=.956	Passed K=1 A=0	Failed	Passed AHAT=4.94 BSQ=2.338	Failed	Failed	Passed P=.003 M=1
4	Passed λ= .00420	Passed K=1	Passed α= 315.24 β=1.05	Passed K=1 A=0	Failed	Failed	Failed	Failed	Passed P=.005 M=1
5	Passed λ= .003864	Passed K=1	Passed α= 226.8 β=.98	Passed K=1 A=0	Failed	Passed AHAT=4.8 BSQ=2.348	Failed	Failed	Passed P=.004 M=1
6	Failed	Failed	Passed α= 94.87 β= .8451	Passed K=1 A=0	Passed K=6.84 A=1393	Passed AHAT=4.47 BSQ=3.34	Failed	Failed	Failed

Table 4.2 (continued)

Cycle	Expo-nential	Erlang	Weibull	Gamma	Pearson XI	Lognormal	Beta	Normal	Negative Binomial
7	Failed	Failed	Failed	Passed K=1 A=0	Passed K=4.05 A=695.38	Passed AHAT= 4.355 BSQ= 2.932	Failed	Failed	Failed
8	Passed λ= .00422	Passed K=1	Passed α= 236.97 β=1	Passed K=1 A=0	Failed	Passed AHAT= 4.754 BSQ=2.185	Failed	Failed	Passed P=.003 M=1
9	Failed	Failed	Failed	Passed K=1 A=0	Passed K=5.19 A= 688.98	Passed AHAT= 4.325 BSQ=1.921	Failed	Failed	Failed
10	Passed λ= .004675	Passed K=1	Passed α= 213.9 β=1	Passed K=1 A=0	Passed K=7.62 A= 1415.55	Passed AHAT= 4.488 BSQ=2.892	Failed	Failed	Passed P=.003 M=1

Note: This table includes the parameters calculated by SIMFIT for the distributions which passed a G-O-F test.

The SIMFIT output for the LN-15 IMU is summarized in Table 4.3. The LN-15 unit failure cycle samples were modeled by the exponential distribution for only one of the four cycles considered. Since the gamma, Weibull, Erlang, and Pearson XI passed two of the four cycles, all four of these distributions were considered better models for the LN-15 unit than the exponential distribution. The analysis of the LN-15 failure data lends support to the research hypothesis.

To help visualize the goodness-of-fit for the failure cycle samples to the theoretical distributions, Tables 4.4, 4.5, and 4.6 were constructed. They provide a list of relative rankings of the theoretical models based on the "pass" or "fail" criteria established in Chapter II. The exponential distribution was <u>not</u> found to be the "best" model for any unit, while the gamma proved to be the best model for the KT-73 and FLIP units. Therefore, in summary, the research hypothesis was supported for all three units.

The ranked comparison of the best model for all failure cycle samples from all three units is presented in Table 4.7. This table shows that the gamma distribution provided the best overall model for the units tested. Fourteen of the twenty failure cycle samples tested fit the gamma distribution. The exponential model passed only six of the twenty failure cycle samples tested for G-O-F by the

Table 4.3

Summary of the SIMFIT Results for the LN-15 Unit

Cycle	Exponential	Erlang	Weibull	Gamma	Pearson XI	Lognormal	Beta	Normal	Negative Binomial
1	Failed	Passed K=3	Passed α= 18092.2 β= 1.7775	Passed K=2 A=.2	Failed	Passed AHAT= 6.285 BSQ=.0363	Passed A=.568 Γ= 4.255	Failed	Passed P=.005 M=3
2	Passed λ= .00423	Passed K=1	Passed α= 445. 0153 β= 1.1079	Passed K=1 A=.1	Failed	Failed	Failed	Failed	Failed
3	Failed	Failed	Failed	Failed	Passed K=7.2 A=7.38	Passed AHAT= 3.971 BSQ= 2.315	Failed	Failed	Failed
4	Failed	Failed	Failed	Failed	Passed K=2.46 A= 105.07	Failed	Failed	Failed	Failed

Note: This table includes the parameters calculated by SIMFIT for the distributions which passed a G-O-F test.

Table 4.4

Ranked Comparison of the Best Model for the KT-73 Unit

Distribution	Number Passed	Total Number	% Passed
Gamma	3	6	50
Lognormal	2	6	33
Weibull	1	6	17
Erlang	1	6	17
Beta	1	6	17
Exponential	0	6	0
Pearson XI	0	6	0
Negative Binomial	0	6	0
Normal	0	6	0

Table 4.5

Ranked Comparison of the Best Model for the FLIP Unit

Distribution	Number Passed	Total Number	% Passed
Gamma	9	10	90
Weibull	7	10	70
Lognormal	7	10	70
Exponential	5	10	50
Erlang	5	10	50
Pearson XI	5	10	50
Negative Binomial	5	10	50
Beta	0	10	0
Normal	0	10	0

Table 4.6

Ranked Comparison of the Best Model for the LN-15 Unit

Distribution	Number Passed	Total Number	% Passed
Gamma	2	4	50
Weibull	2	4	50
Erlang	2	4	50
Pearson XI	2	4	50
Lognormal	2	4	50
Exponential	1	4	25
Negative Binomial	1	4	25
Beta	1	4	25
Normal	0	4	0

Table 4.7

Ranked Comparison of the Best Model for All Units

Distribution	Number Passed	Total Number	% Passed
Gamma	14	20	70
Lognormal	11	20	55
Weibull	10	20	50
Erlang	8	20	40
Pearson XI	7	20	35
Exponential	6	20	30
Negative Binomial	6	20	30
Beta	2	20	10
Normal	0	20	0

SIMFIT program; therefore, the exponential distribution was not the best model for the failure cycles of the three IMUs.

Data aggregation results. After testing failure cycles for underlying distributions, the SIMFIT results obtained from the aggregated data samples were analyzed. The aggregated data from each of the three IMU populations did not fit any of the theoretical distributions tested by SIMFIT. This result was somewhat surprising because individual failure cycle samples were able to fit at least one of the distributions tested for sixteen of the twenty individual cycle samples.

Currently, the Air Force aggregates failure data in most data collection systems. The assumption that failures follow an exponential distribution, which was solidified years ago by military standards and regulations that supported the "exponential assumption," is the primary reason that data collection systems currently aggregate data. The SIMFIT results obtained for the aggregated IMU failure data would seem to indicate that a data collection system, based on the aggregation of failure data, is nearly useless in trying to discover (or even verify as for the exponential) an underlying distribution for the failure data. If this underlying distribution is not obtainable, then realistic prediction of operating characteristics of components/

systems and of logistical support requirements cannot be made.

Convoluted sampling results. In continuing the analysis of the IMU failure data, SIMFIT was used to test randomly drawn convoluted samples from each of the three populations. Although this analytical procedure would result in only a negative-type of test; i.e., the exponential assumption could only be disproved (as discussed in Chapter III), its inclusion was deemed necessary as an alternate source of possible support for the research hypothesis.

The convoluted data for the KT-73 unit failed the SIMFIT criteria of passing either or both G-O-F tests for all theoretical tested except for the normal distribution. Therefore, since the convoluted data were not able to fit the Erlang distribution, the underlying failure model could not have been the exponential distribution. This finding supported the research hypothesis.

The convoluted data for both the FLIP unit and the LN-15 unit passed the Erlang distribution. The fact that the convoluted data were able to fit the Erlang model does not, in itself, substantiate the assumption that the underlying distribution was exponential for the individual failure cycles of these two units; therefore, this outcome does not contradict earlier support for the research hypothesis.

The convolution data samples used for this analysis are contained in Appendix E and their SIMFIT histograms are contained in Appendix F.

Reliability comparison results. The final phase of the planned analysis entailed reliability comparisons between contract specified reliability and achieved reliability computed for the distribution parameters provided by SIMFIT. One extremely important reason for accurately identifying the underlying distribution of a sample of failure data is to make possible the use of established parametric statistical methods and techniques to further characterize the sample data.

The sample parameters for each of the failure cycles were obtained from SIMFIT. The means, standard deviations, variances, and the number of failures observed for each cycle are tabularized in Table 4.8.[2] After observing the means in this table, the mean time between failures appeared to be much less for all three units after the first failure than the mean time to failure of the first cycle.

For instance, the MTBF given in the contract for the LN-15 unit was 600 hours (18); however, the value of

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[2]An interpretation of these statistics must be made with the cognizance that some units may still be operating as of the data collection cutoff date while others may have been discarded for repeated failures or may be somewhere within the supply channel.

Table 4.8

Table of Sample Parameters

Unit	Mean	Standard Deviation	Variance	# of Failures
KT-73				
Cycle 1	818.06	539.41	290963.15	554
Cycle 2	258.33	306.92	94199.89	451
Cycle 3	226.85	286.88	82300.13	354
Cycle 4	165.83	216.34	46803.00	258
Cycle 5	183.27	256.56	65823.03	185
Cycle 6	135.83	209.98	44091.60	120
FLIP				
Cycle 1	900.95	709.90	503958.01	145
Cycle 2	283.90	490.56	240649.11	136
Cycle 3	300.43	314.15	98690.22	127
Cycle 4	233.31	222.14	49346.18	124
Cycle 5	258.84	264.94	70193.20	118
Cycle 6	238.61	283.67	80468.67	110
Cycle 7	228.34	321.13	103124.48	96
Cycle 8	236.82	262.68	69000.78	82
Cycle 9	164.31	209.54	43907.01	62
Cycle 10	213.90	249.08	62040.85	49
LN-15				
Cycle 1	635.07	369.17	136286.49	316
Cycle 2	236.54	213.92	45761.77	213
Cycle 3	119.01	140.02	19605.60	117
Cycle 4	72.18	167.58	28083.06	49

the MTBF for all cycles after the first cycle was less than the 600 hours specified. After renewal the KT-73 unit, with a contract specified MTBF of 650 hours (33), also had a decreased MTBF. Therefore, the assumption of complete renewal apparently was not valid for these units.

After renewal, the actual reliability obtained for all cycles was less than the specified reliability. Figure 4.1 shows the reliability plotted for parameters specified in the contract for the LN-15. Figure 4.2 shows the reliability plotted for the parameters obtained from SIMFIT for the second failure cycle using the exponential distribution. From these graphs it can be observed that the actual reliability was considerably less than that given by the contract specification.

For example, the probability of a LN-15 unit operating 600 hours using the contract specified reliability is 37 percent. The probability of the unit actually operating 600 hours after the first renewal is only 7 percent. These points are represented by the dotted lines on Figures 4.1 and 4.2. The same general pattern was found present for the KT-73 unit. Both the KT-73 and LN-15 units exceeded the contractor specified reliability for the first failure cycle, but after renewal the reliability dropped below these specified levels.

Although no contract specified reliability was obtained for the FLIP unit, a decrease in reliability was

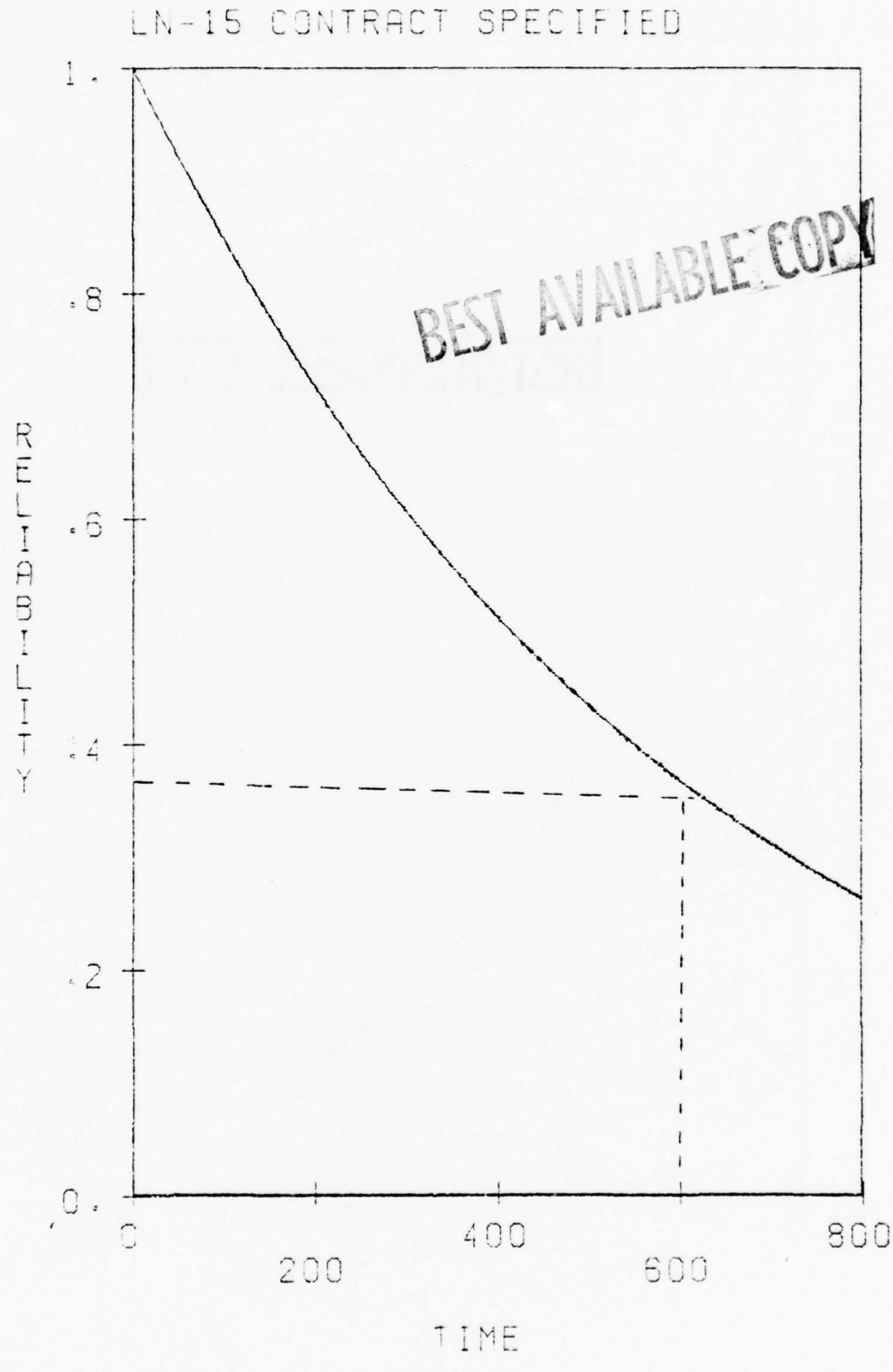

LN-15 CONTRACT SPECIFIED
1.
.8
.6
.4
.2
0.
RELIABILITY
0
200
400
600
800
TIME

Figure 4.1

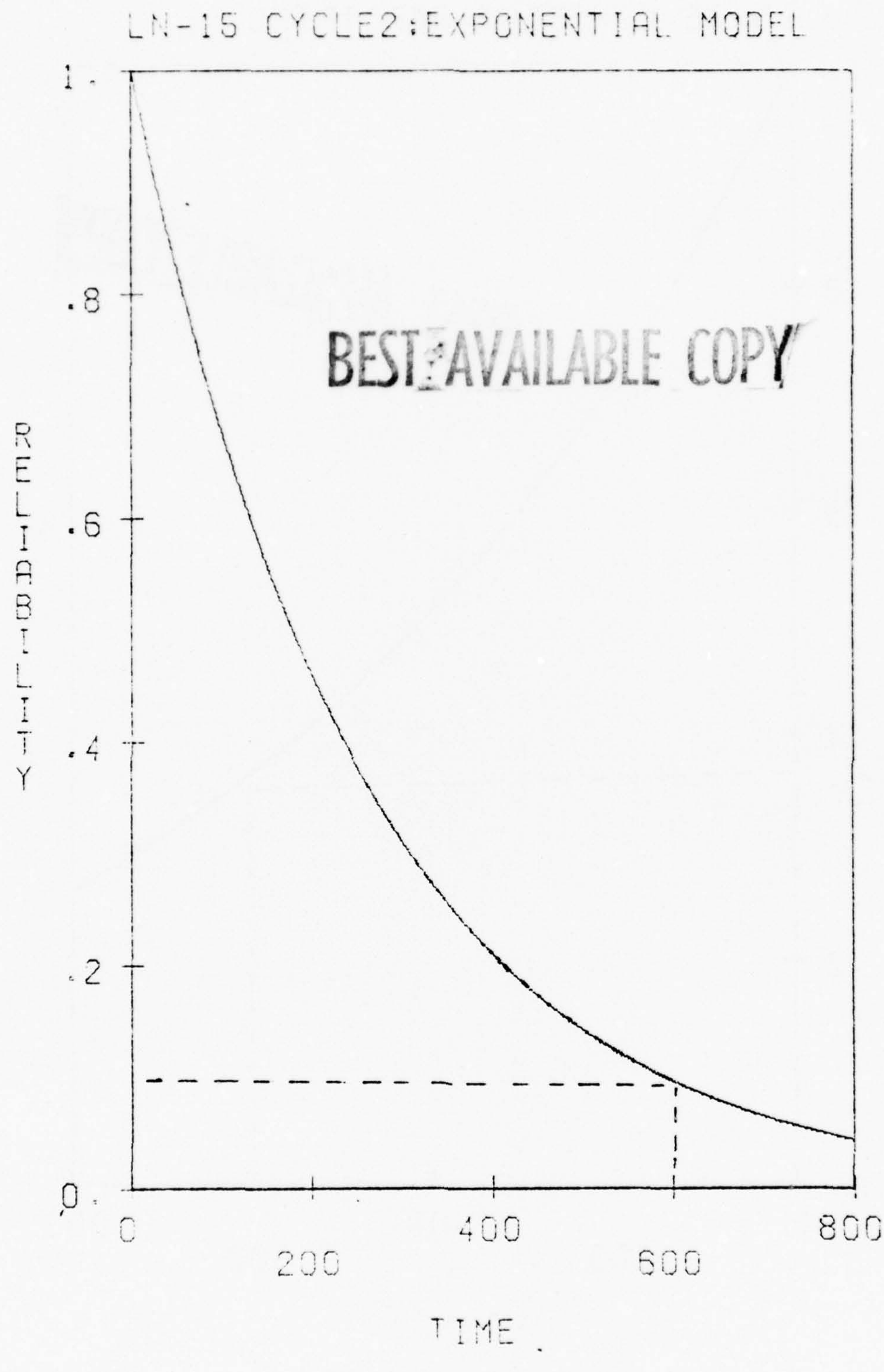

LN-15 CYCLE2:EXPONENTIAL MODEL
RELIABILITY
1.
.8
.6
.4
.2
0.
0
200
400
600
800
TIME

Figure 4.2

evident after renewal. Figures 4.3, 4.4, and 4.5 of the Weibull model and Figures 4.6 and 4.7 of the exponential model were used to graphically display this decrease in reliability. The model selected, in each case, had the smallest maximum error for the K-S G-O-F test calculated by SIMFIT for each individual failure cycle.

The hazard rates obtained for the units were also graphed to emphasize that the actual hazard rate increased after renewal and to show that the hazard rates calculated from the SIMFIT outputs were not constant from cycle to cycle. The graphs of the hazard rates of the units show an increasing hazard rate for the first failure cycle and a decreasing hazard rate for units after renewal.

For example, Figure 4.8 shows an increasing hazard rate for the first cycle of the FLIP unit using the Weibull model. The Weibull model had the smallest maximum error in the K-S G-O-F test for this cycle, and therefore, was chosen to graph the hazard rate. Figure 4.9 shows a decreasing hazard rate for cycle three of the FLIP unit using the Weibull model. These hazard rate graphs clearly show that the hazard rate was higher for cycle three failure data than for cycle one. The same general pattern was present for the KT-73 and LN-15 units.

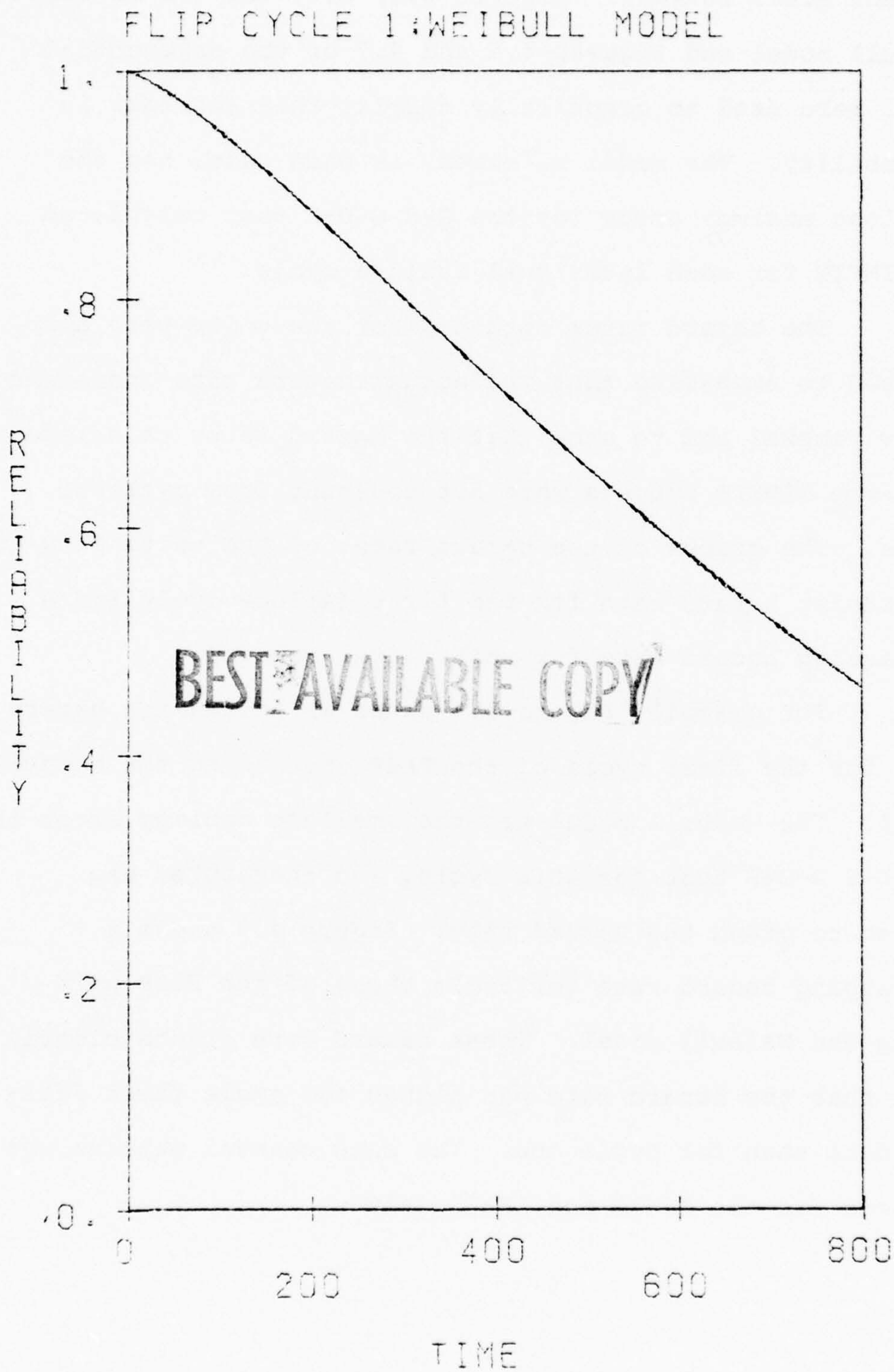

FLIP CYCLE 1:WEIBULL MODEL
1.
.8
.6
.4
.2
.0
RELIABILITY
0
200
400
600
800
TIME

Figure 4.3

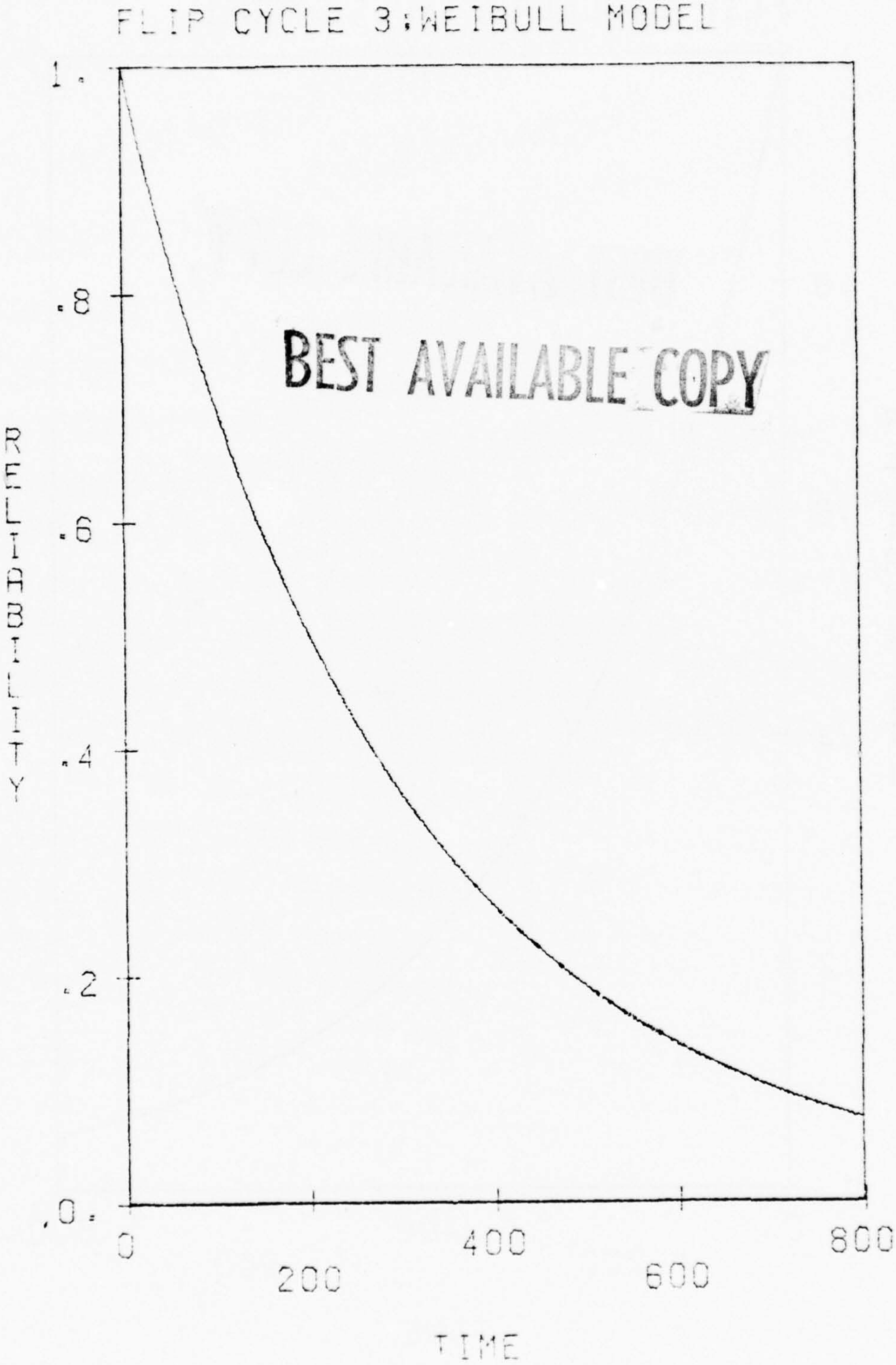
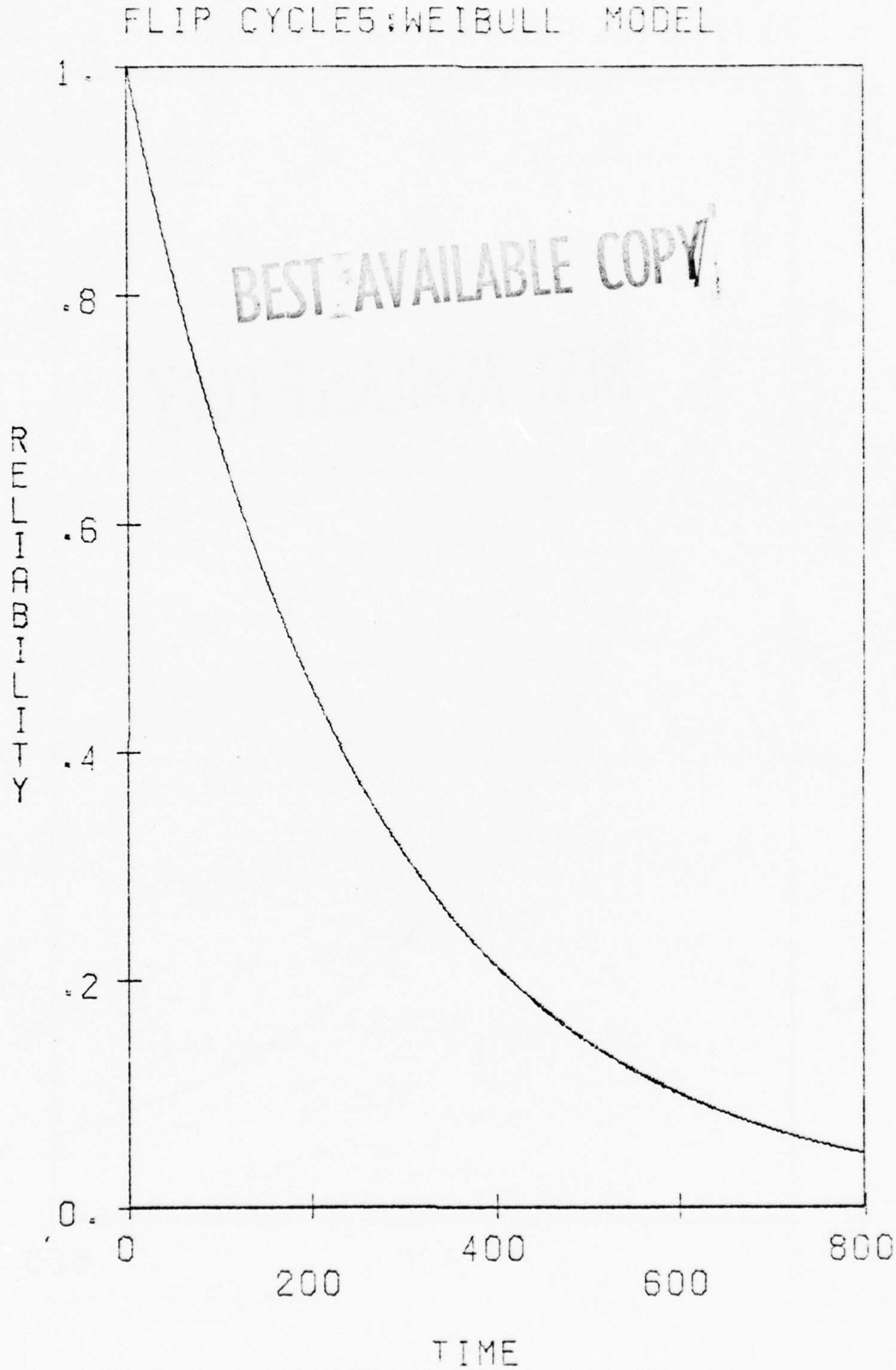

FLIP CYCLE 3;WEIBULL MODEL
1.
.8
.6
.4
.2
.0
RELIABILITY
0
200
400
600
800
TIME

Figure 4.4



FLIP CYCLES:WEIBULL MODEL
RELIABILITY
1.
.8
.6
.4
.2
0.
0
200
400
600
800
TIME

Figure 4.5

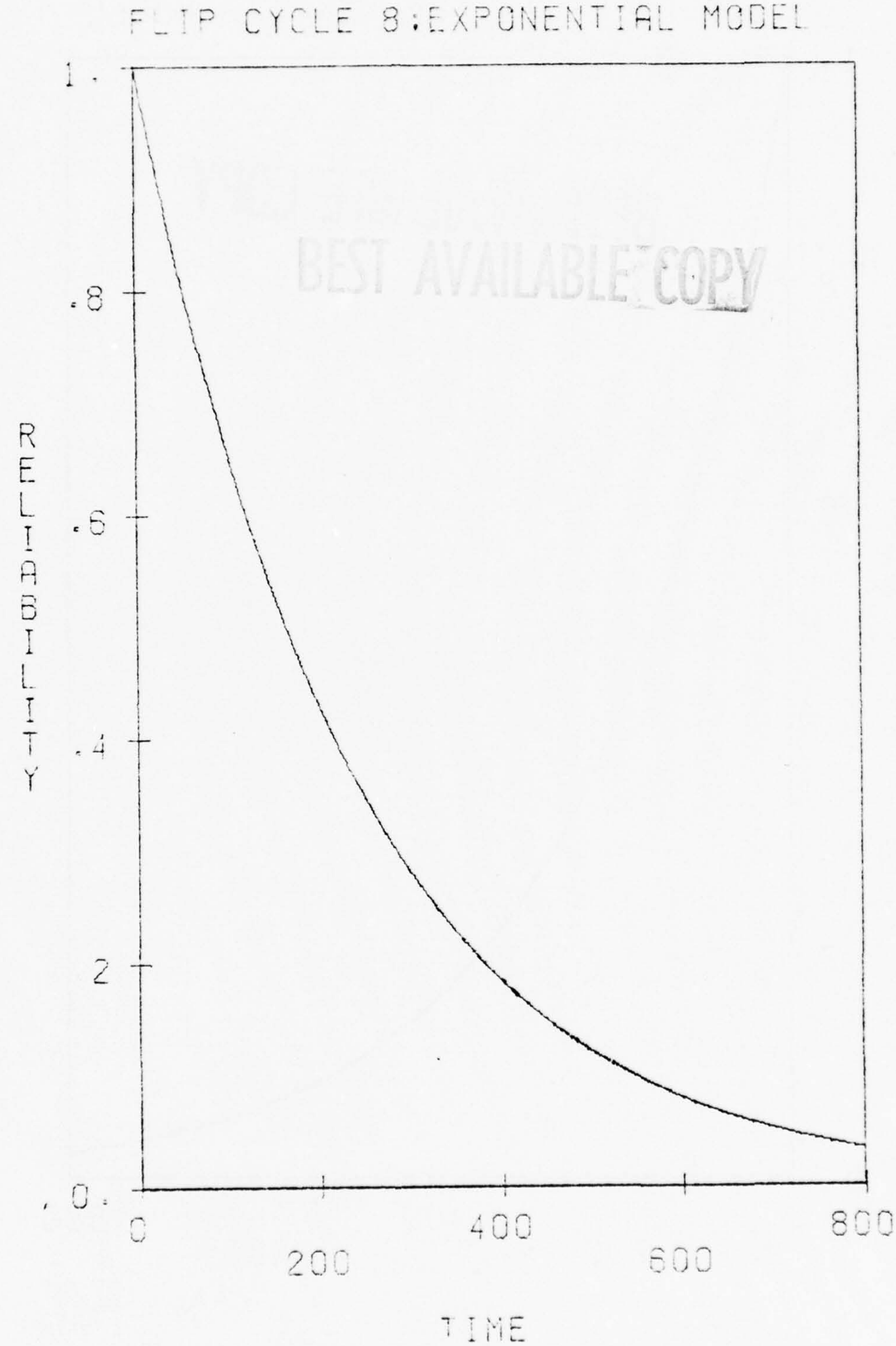
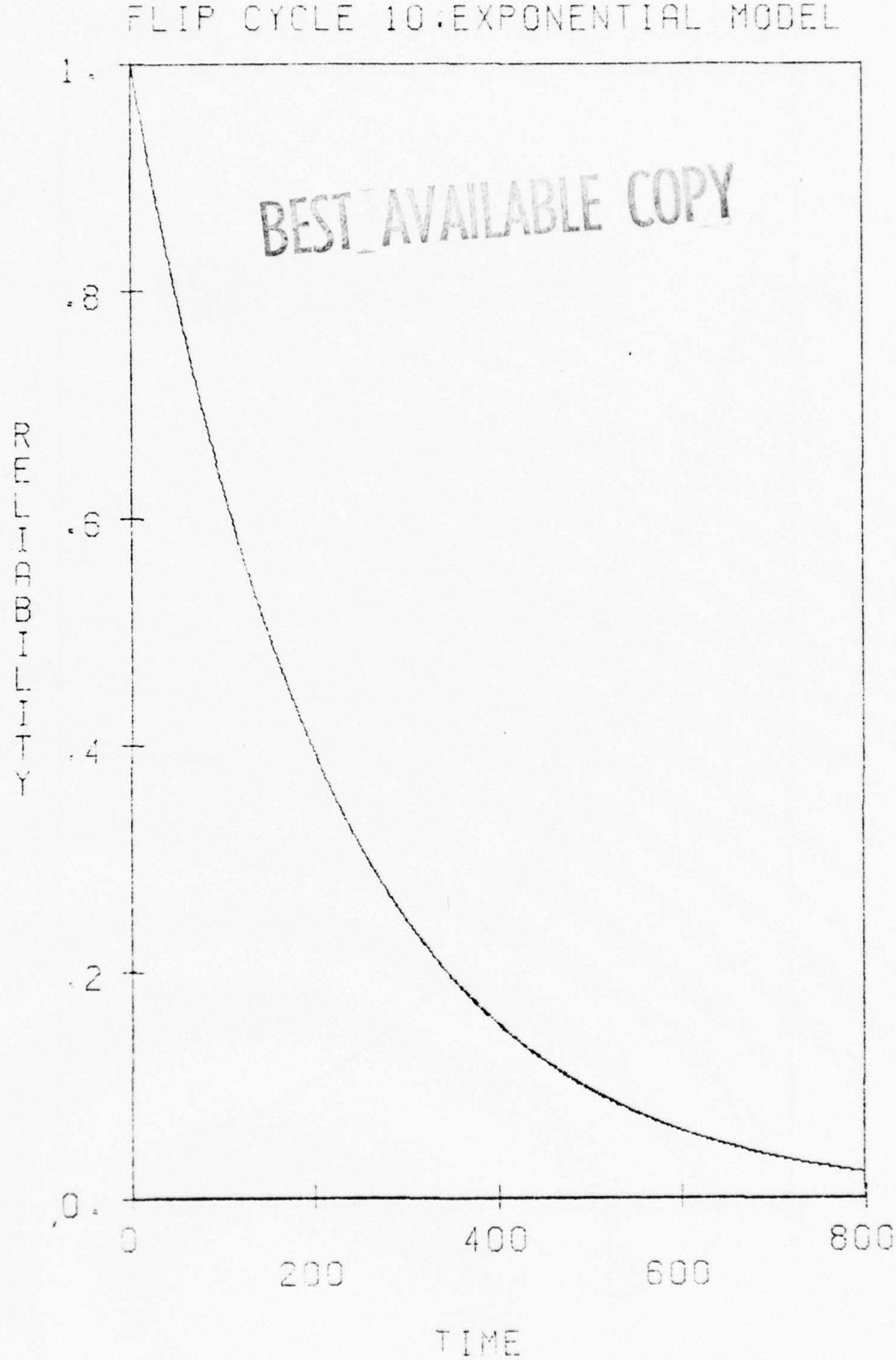
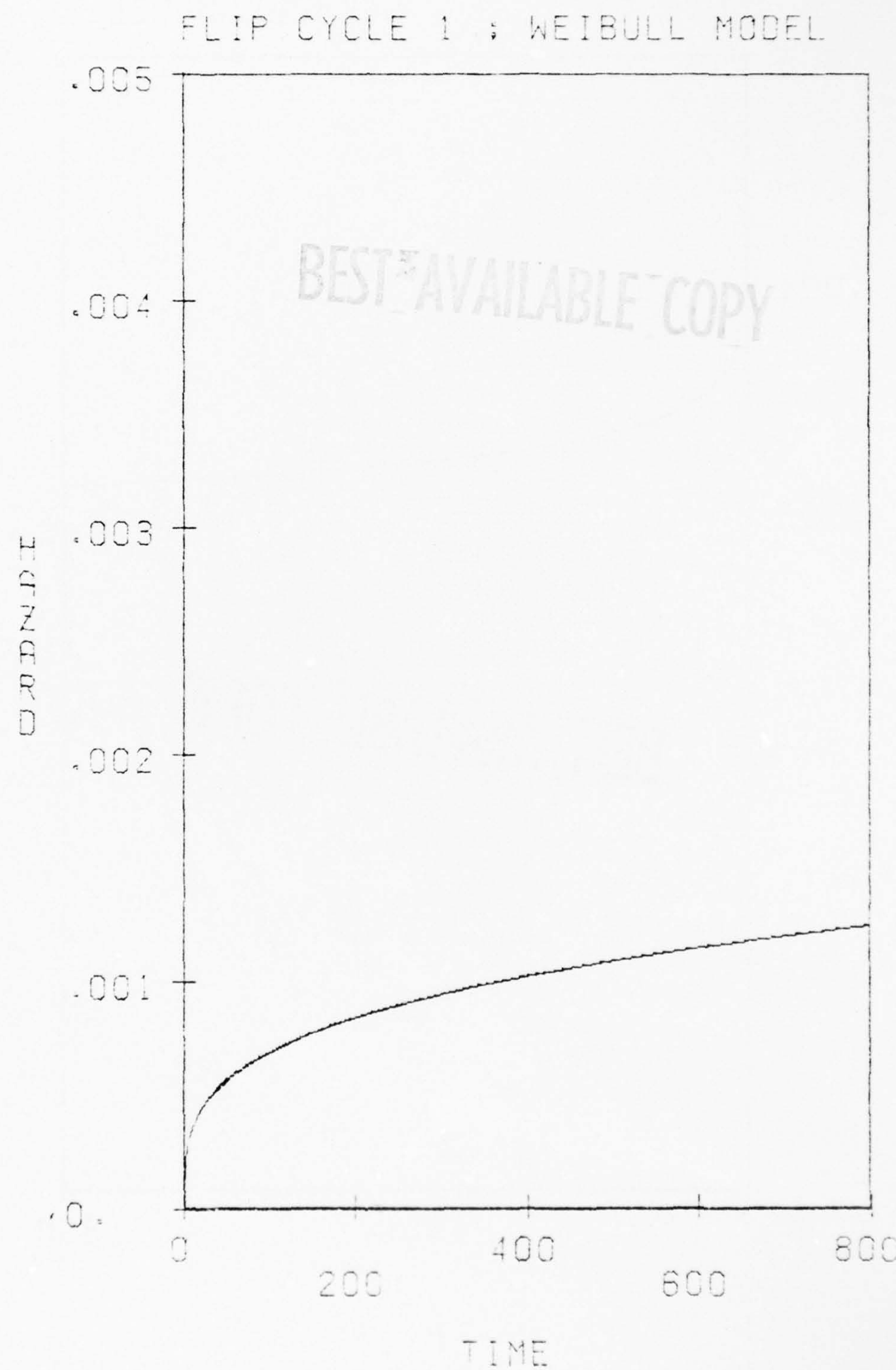

FLIP CYCLE 8:EXPONENTIAL MODEL
1.
.8
.6
.4
2
.0.
RELIABILITY
0
200
400
600
800
TIME

Figure 4.6



FLIP CYCLE 10:EXPONENTIAL MODEL
RELIABILITY
1.
.8
.6
.4
.2
0.
0
200
400
600
800
TIME

Figure 4.7



FLIP CYCLE 1 ; WEIBULL MODEL
.005
.004
.003
.002
.001
.0
HAZARD
0
200
400
600
800
TIME

Figure 4.8

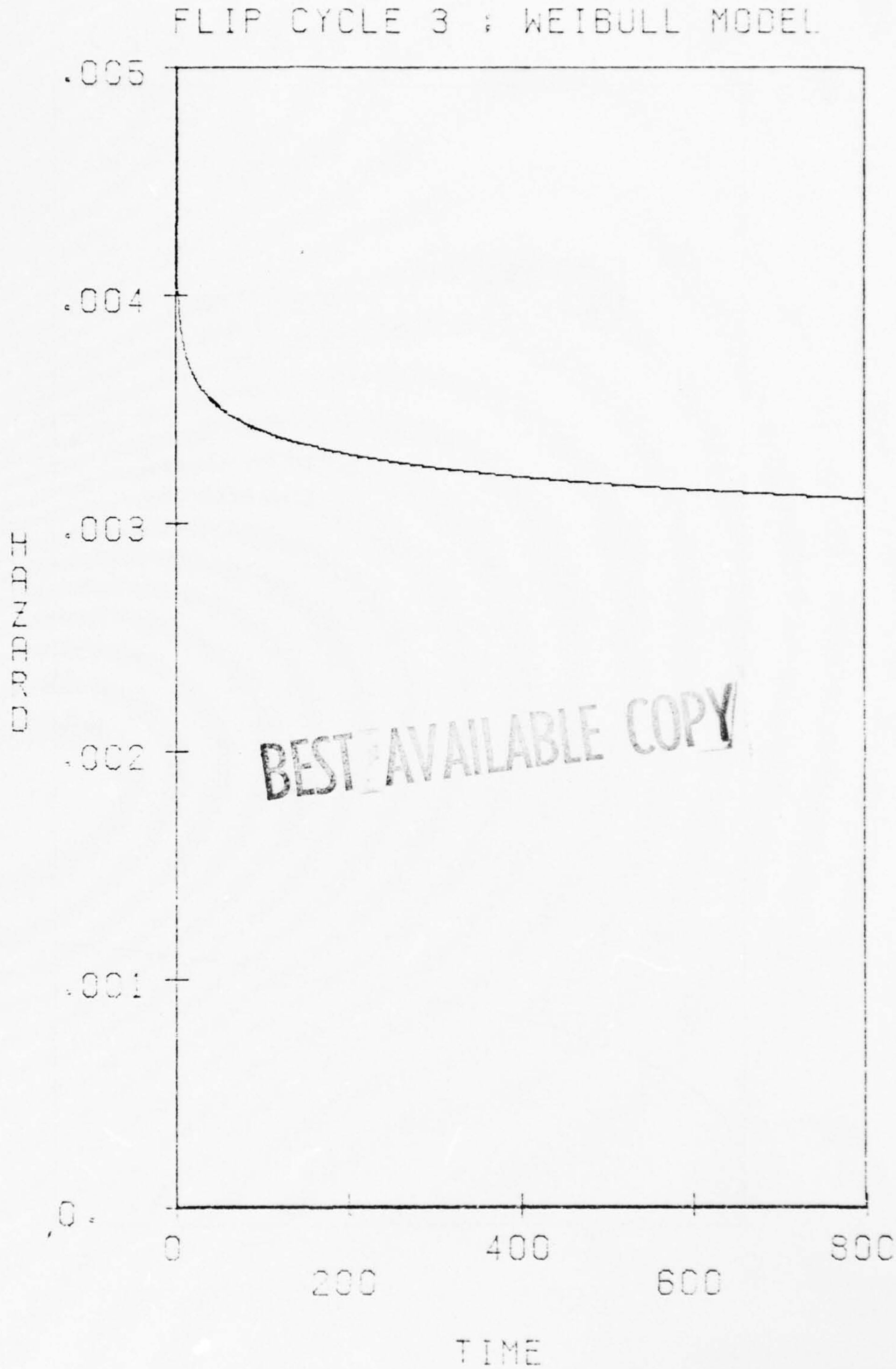

FLIP CYCLE 3 ; WEIBULL MODEL
.005
.004
.003
.002
.001
.0
HAZARD
0
200
400
600
800
TIME
BEST AVAILABLE COPY

Figure 4.9

Further Analysis

During many research efforts, unexpected findings are encountered. At times these unexpected results are more interesting and valuable than the original research objective. Therefore, the unexpected results are presented without full knowledge of their significance and with the hope that these findings will aid future research efforts.

Unexpected results. The failure data for each failure cycle of the FLIP unit (except cycle two) and the LN-15 unit passed at least one distribution when compared with the distributions checked. The failure data on the KT-73 unit for cycles two, three, and four, however, did not pass any of the distributions tested. These three failure cycles were further analyzed. The SIMFIT histogram cells sizes were set equal and the distributions truncated by 'cutting off' the cells which had fewer than two entries on the histogram. Figure 4.10 resulted from this procedure. This histogram is for failure cycle three but similar histograms were obtained from cycles two and four. The peaks and valleys of the histogram explain why these distribution would not fit any of the theoretical distributions tested. SIMFIT contains no theoretical distribution with these cyclíc peaks and valleys; therefore, further analysis was not feasible in this research.

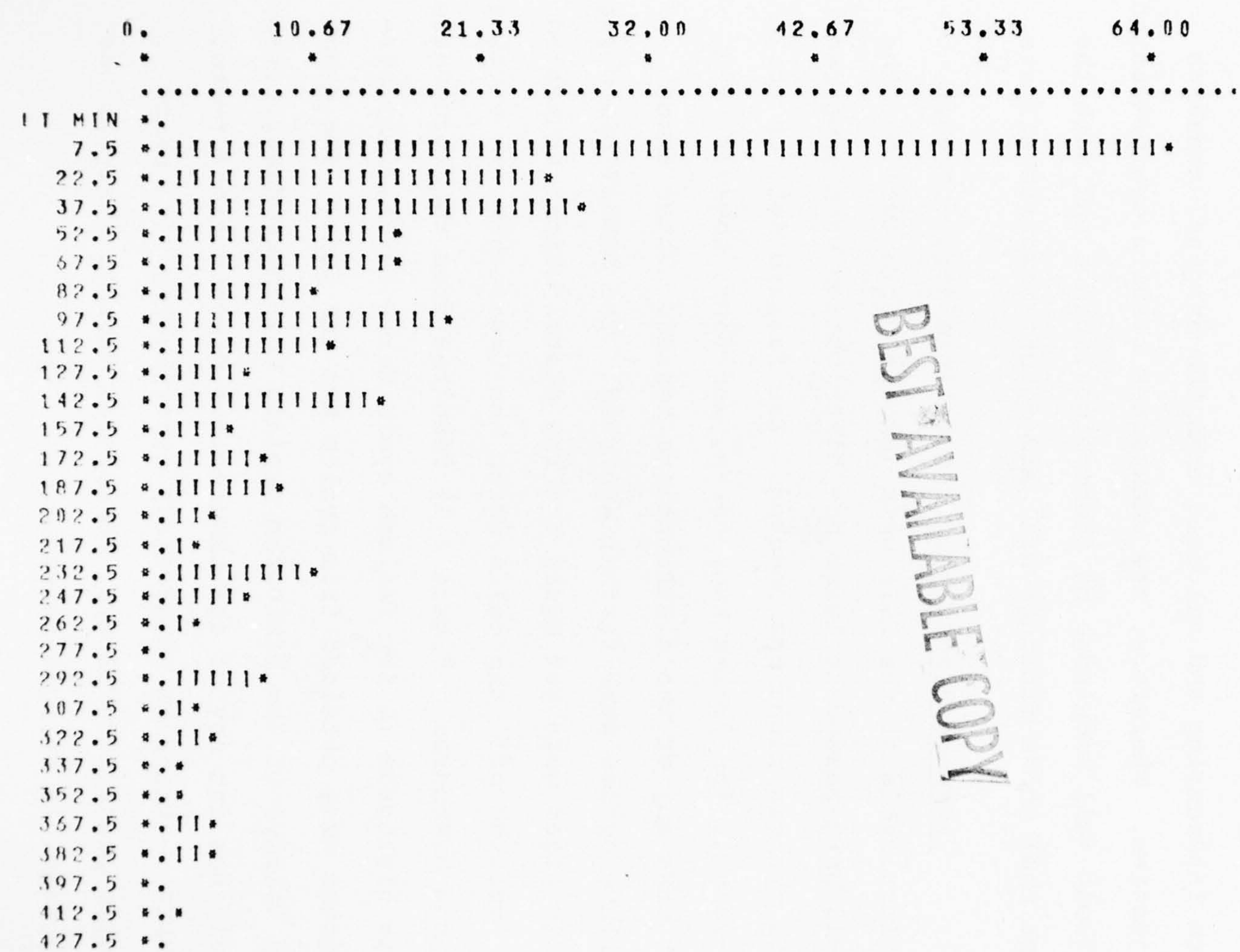


Figure 4.10. Histogram of Failure Cycle 3
for the KT-73 Unit

Infant mortality. A second interesting finding was a large infant mortality. A review of the SIMFIT histograms in Appendix F indicated that a large number of early failures (infant mortality) had occurred for renewed units. The G078C Data Collection System defines "zero timers" for these three units as a failure that occurs within the first 15 hours of operation. This value applies to either new units or renewed units (29). Therefore, failures which occurred within 15 or less hours after renewal were considered as cases of infant mortality. These were enumerated and compared to the total number of failures for each failure cycle. The results are summarized in Table 4.9. Each unit had a large infant mortality after renewal which resulted in the large peak in cell one of the histograms for all cycles after the initial cycle. The large percentage of infant mortality suggests that the exponential distribution should not be used to model the failures of renewed IMUs (3:70-5). Infant mortality is normally associated with electronic components and therefore was not expected for these units.

Distribution shift. After reviewing Tables 4.1, 4.2, and 4.3, it was clear that renewed units did not necessarily follow the same distribution from cycle to cycle. The LN-15 IMU, for example, passed the Erlang, Weibull, and gamma for cycles one and two, but passed the

Table 4.9

Infant Mortality

Unit	Unit Cycle	Number of Failures	Number of Failures In Less 15 Hours	% of Failures in Less than 15 Hours
KT-73	1	554	1	.18
	2	451	66	14.60
	3	354	68	19.20
	4	258	53	20.50
	5	185	37	20.00
	6	120	42	35.00
FLIP	1	145	1	.69
	2	136	22	16.2
	3	127	1.0	7.87
	4	124	14	11.29
	5	118	14	11.86
	6	110	19	17.27
	7	96	19	19.70
	8	82	7	8.54
	9	62	8	12.90
	10	49	8	16.33
LN-15	1	316	0	0
	2	213	17	7.98
	3	117	28	23.93
	4	49	19	38.76

Pearson XI and the lognormal distribution for cycle three, and subsequently passed only the Pearson XI on cycle four failures. This "distribution shift" was also apparent in the KT-73 and FLIP inertial measurement units.

Two possible reasons for the change in the distributions from cycle to cycle of the same population are:

1. The units with long operating lives have not yet failed and when these units fail the distribution will shift back to the same distributions as found in the earlier cycles.

2. Due to the aging process different sub-units are failing in the later cycles changing the underlying distributions describing the pattern of failures.

If this "distribution shift" is present in many complex systems which are renewed, then knowledge of the time and pattern of these shifts would be a powerful planning tool.

Summary

The analysis of the SIMFIT results indicated that the exponential distribution was not the best model for the underlying distribution of the failure data from the three IMU populations tested. The gamma distribution proved to be the "best" model for these units. Since other distributions were better models for the IMU failure data than

the exponential distribution, the research hypothesis was supported.

The data aggregation analysis indicated that valuable information was lost by the aggregation process. While the individual failure cycle samples passed certain distributions, this information about the failure models was lost when the data were aggregated.

The analysis of the convolution samples were somewhat inconclusive. The KT-73 data did not fit the Erlang distribution when convoluted; therefore, the exponential distribution was not an appropriate model to describe failures for that population. Inasmuch as the data for the other two IMUs (LN-15 and FLIP) passed the Erlang distribution; the exponential distribution could not be ruled out as being the possible underlying distribution of the failure data based only on the results obtained from the convolution analysis.

After renewal, the reliability of the units was less than that which was specified in the contracts for the LN-15 and KT-73 IMUs. The assumption of complete renewal with the exponential model being the underlying distribution of failures after renewal does not appear correct, because the "exponential assumption" indicates that there is no decrease in the reliability of a specific unit even after 100 renewals.

Several unexpected results surfaced during the analysis of the failure data. The high infant mortality, after renewal, indicated that the exponential model was inappropriate for these units. An additional fact was that the failure distributions were not constant from cycle to cycle. No generalization was possible as to the exact reason for this "distribution shift" between cycles. This shift could be peculiar to the units tested or could be a general failure pattern for many types of units.

CHAPTER V

CONCLUSIONS AND RECOMMENDATIONS

This chapter discusses the conclusions derived from the analysis of failure data for the three Inertial Measurement Units. First, the research hypothesis is addressed. Then, additional conclusions are drawn based on the analysis of the failure data. The chapter ends with a presentation of recommended areas for future research.

Support for the Research Hypothesis

The research hypothesis was stated as: The distribution of failure for some replaceable/repairable units follow a statistical distribution which differs from the exponential distribution model. All three populations of IMU failure data that were analyzed provided support to this research hypothesis. For the FLIP and LN-15 units the gamma proved to be the best fit, and for the KT-73 unit four failure distributions proved to be better fits than the exponential model. These results supported the research hypothesis. In addition, the convoluted data for the KT-73 unit did not fit the Erlang distribution which indicated that the exponential model was inappropriate. Once again the research hypothesis was supported.

The gamma distribution was able to fit a total of fourteen of the sample failure cycles out of the twenty

tested. This result suggested that the gamma model may be a better failure model for all Inertial Measurement Units than the assumed exponential model. Of course, this result should be verified by additional analysis of other IMUs. If this conclusion is verified, then the Air Force should consider the use of the gamma model when contracting for additional IMUs.

Although the gamma distribution was able to model many of the individual failure cycles, when data was aggregated, the gamma distribution did not fit the data. This data aggregation provided expected, though disappointing, results. None of the three populations passed any distribution tested after the failure data were aggregated. This conclusion supported the argument against aggregation of failure data.

Currently most data collection systems within the Air Force aggregate data. This data is then used to calculate the MTBF by the formula,

$$\text{MTBF} = \frac{\text{total operating hours}}{\text{total number of failures}} .$$

This equation is valid only when using the exponential model, because after data have been aggregated, the total operating hours and the total number of failures are the only information which is available. Therefore, the

exponential model must be assumed because other possible models require more information than is provided by aggregated data.

Renewal

The analysis of the failure data showed that the concept of complete renewal was not valid. There was a significant drop in operating times between failures in the first cycle and failures in the second cycle. This large drop could have been caused by two primary factors.

1. The units were operated and repaired by the contractor before the Air Force gained repair responsibility for the units.

2. The elapsed time indicators were not set to zero when the Air Force acquired the units. For example, a type of "burn in" could have been employed by the contractor.

The complete renewal concept was also shown to be inappropriate because of the high infant mortality for renewed units. For example, failure cycle four of the LN-15 unit had a 39% infant mortality rate. This high infant mortality could have been caused by design imperfections in each of the three units or by improper or incomplete maintenance and/or checkout of the units. This research could not determine the cause for the infant mortality, but clearly the concept of complete renewal was inappropriate for these IMUs.

Recommendations

In any research effort many interesting and perplexing questions are encountered which cannot be completely resolved. Besides the required amount of time being prohibitive, the research process itself continually regenerates new and just as perplexing questions. Therefore, many questions arising from a research effort must be left for future research. The following list, in varying detail, are areas relating to this thesis that are recommended for future research.

1. One area of recommended research, which might provide the most significant impact in relation to this thesis, would be an analysis between the cost actually incurred by assuming the exponential failure model and the cost that would have been incurred if the gamma model had been used in planning. This research could be approached by a cost analysis of one or more of the three IMUs, and then by the use of computer simulation to complete the analysis.

2. An investigation should be initiated to discover why "distribution shifts" occur between failure cycles for an IMU. One approach would be to obtain more failure data for these IMUs, whenever they become available, and analyze these data. If the "distribution shifts" remain, more detailed failure data from cards 2-5 of the G078C Data

Collection System could be obtained and analyzed as a means of trying to identify specific sub-unit failure patterns. These patterns may be influential in causing the observed "distribution shifts" in the IMUs.

3. Since research builds on research to provide insights into reliability, it is recommended that another set of IMU failure data be collected and analyzed to (a) replicate this thesis and (b) determine if a generalization concerning the failure patterns for inertial measurement units can be made.

4. A recommendation is made to investigate any change in the distribution for the FLIP unit after failure data has been collected on the unit's modification which began in May 1976. If the underlying failure distribution does change as a result of the modification, the information would prove valuable to future modifications of similar IMUs.

5. Future research is recommended to determine if other electro-mechanical units experience the same high infant mortality after renewal as that which is experienced by the IMUs in this research.

6. A research effort, similar to the one performed by this thesis, is recommended to establish if support for the research hypothesis can be achieved using non-aggregated data of the G097C Data Collection System (16).

7. Another recommendation for future research entails the use of other distributions than those contained within SIMFIT to determine if the failure data for the three IMUs can be better modeled with a distribution other than the gamma.

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APPENDIX A

APPLICABLE AIR FORCE
RELIABILITY DOCUMENTS

Air Force Regulation 80-5, *Reliability and Maintainability Programs for Systems, Subsystems, and Equipment and Munitions*.

MIL-HDBK-217B, *Reliability Prediction of Electronic Equipment*.

MIL-STD-1050, *Sampling Procedures and Tables for Inspection by Attributes*.

MIL-STD-414, *Sampling Procedures and Tables for Inspection by Variables for Percent Defective*.

MIL-STD-690A, *Failure Rate Sampling Plans and Procedures*.

MIL-STD-721B, *Definition of Effectiveness Terms for Reliability, Maintainability, Human Factors, and Safety*.

MIL-STD-756A, *Reliability Prediction*.

MIL-STD-757, *Reliability Evaluation from Demonstration Data*.

MIL-STD-781B, *Reliability Tests: Exponential Distribution*.

MIL-STD-785, *Requirements for Military Programs (for Systems and Equipment)*.

MIL-STD-790A, *Reliability Assurance Programs for Electronic Parts Specification*.

MIL-STD-1235, *Sampling Procedures and Tables for Continuous Inspection by Attributes*.

TR-7, *Factors and Procedures for Applying MIL-STD-105D Sampling Plans to Life and Reliability Testing*.

APPENDIX B

SIMFIT COMPUTER PROGRAM

The SIMFIT computer program is used to test a distribution to determine if a given sample follows a particular theoretical or hypothesized distribution. The SIMFIT program calculates the parameters needed to determine the probability distribution from the input data. The input data are divided into cells and compared to the theoretical value for each cell. Both the nonparametric K-S one sample test and the Chi-square test are used to test the data at a given confidence interval. The SIMFIT program has the capability of testing the distribution at α = .1, .5, or .01 (29).

The K-S analysis in the SIMFIT program is capable of testing one or all of the twelve distributions contained within the SIMFIT repertory. These distributions are: Erlang, normal, lognormal, gamma, Pearson XI, Weibull, uniform, beta, triangular, Poisson, negative binomial, and positive binomial. The Erlang with parameter k = 1 is the equivalent to the exponential distribution and the SIMFIT program was used to test this distribution as well as the Erlang, normal, lognormal, gamma, Pearson XI, Weibull, beta, and negative binomial in this research analysis.

SIMFIT presents a Chi-square analysis in addition to the K-S analysis. The Chi-square statistic is only computed for that portion of the distribution in which the cell size criteria is met.

The SIMFIT program also constructs a histogram from the input data. The theoretical distribution is compared to the input data on this histogram. The user is required to input the scale, minimum data value, width of each cell, and the number of cells in order to effectively use the histogram. To obtain the best results these values were approximated by the following formulas:

Number of Cells = (number of failure data)(.3) + .5.

Range = maximum value of data—minimum value of data.

Width of Cell = range/number of cells.

If these formulas are not used and some other values are input for the histogram, then the computer program will calculate these values using the above equations and suggest that these calculated values be used if the sample data does not fit the original probability function (30:17).

The SIMFIT program gives the user the option of specifying the parameters of the distribution under test, instead of using the parameters calculated from the data. This option does not assure the user of making the correct decision, but the capability of specifying parameters allows the user to reject a distribution because of incorrectly given or assumed parameter(s).

The SIMFIT program recently received an update by the RAND Corporation. Further anticipated future updates of additional distribution capability will make the SIMFIT program an even more effective tool for the analysis of raw or simulated data.

APPENDIX C

A DISCUSSION OF THE KOLMOGOROV-SMIRNOV AND CHI-SQUARE GOODNESS-OF-FIT TESTS

NONPARAMETRIC METHODS

The two general classes of significance tests are the parametric and nonparametric. Parametric tests are more powerful and are generally the tests used if the associated assumptions are reasonably met. These assumptions include

1. The observations must be independent. That is, the selection of any one case should not affect the chances for any other case to be included in the sample.

2. The observations should be drawn from normally distributed populations.

3. These populations should have equal variances.

4. The measurement scales should be at least interval [9:380].

Nonparametric tests have fewer assumptions. Probably the most important attribute is the lack of a need to assume that the specified population is normally distributed. Also, in nonparametric tests the equal variance assumption is unnecessary. The remaining two assumptions are of varying importance in nonparametric testing, depending upon which test is selected for the given circumstances. The nonparametric statistical test does not require the model

used to specify conditions about the "parameters" of the population from which it has been drawn.

Normally, the fewer and weaker assumptions that constitute a given model, the more general are any conclusions derived from the application of the statistical test associated with that model, but the less powerful is the test of the null hypothesis, H_o. A statistical test is a good one, if it has a small probability of rejecting H_o when H_o is true (small α error), but a large probability of rejecting H_o when H_o is false ($1-\beta$), which is sometimes called the power of a test (27:18-20). This can more readily be visualized by the following illustration.

	State of Nature	
Decision	H_o is true	H_o is false
Accept H_o	Correct Decision	Type II Error (β)
Reject H_o	Type I Error (α)	Correct Decision ($1-\beta$) → Power

The parametric tests have greater power efficiency when their use is appropriate, although some nonparametric tests achieve a power efficiency as high as 95 per cent. This, in essence, means that the nonparametric test can provide the same statistical testing power with a sample of 100 as a parametric test with a sample of 95 (9:380).

If the underlying distribution of given observations of failure times can be determined, more precise estimates of reliability parameters can be obtained. The tools used to do this statistically are called goodness-of-fit tests. Several goodness-of-fit tests can be applied to test a set of failure times for determining whether a hypothesized distribution is a reasonable model or algorithm (30:F-4). Two nonparametric goodness-of-fit tests used by the SIMFIT computer program are the Kolmogorov-Smirnov and the Chi-square goodness-of-fit tests.

KOLMOGOROV-SMIRNOV TEST

The Kolmogorov-Smirnov (K-S) test is a nonparametric goodness-of-fit test used to test a sample to determine if it could have been selected from a particular population distribution. The importance of the K-S test to reliability is in knowing the distribution of random variables (failures or failure times) in order to apply the appropriate mathematical models for analysis of "stochastic" systems (24:209).

The development of the K-S statistic is relatively straightforward. Assume that a given population has a cumulative distribution function F(x) and that a random sample $x_1 \leq x_2 \leq x_3 \leq \cdots \leq x_n$ of size n has been drawn from the population. The empirical cumulative distribution function

$F_n(x)$ is then defined as

$$F_n(x) = \begin{cases} 0 \text{ for } x \leq x_1 \\ \frac{k}{n} \text{ for } x_k \leq x \leq x_{k+1} \\ n \text{ for } x_n \leq x \end{cases}$$

where k is an integer element between 0 and n.

The distance between the two functions is used as a measure of the goodness-of-fit. The maximum vertical deviation is expressed as

$$D = \max_{-\infty<x<+\infty} |F_n(x) - F(x)|$$

Kolmogorov and Smirnov (1; 2; 23) have shown that this vertical deviation D is a distribution-free statistic; i.e., D is a random variable whose distribution function is independent of the underlying distribution function F(x). This distribution-free statistic is the reason that the K-S test for goodness-of-fit is used by the SIMFIT program. The cumulative distribution function, $K_n(x)$, is expressed by

$$K_n(x) = P(D \leq x)$$

is computed as $P(D \leq x) = (1 - \alpha)$ for $n \leq 35$ and as an approximation for $n > 35$ by

$$\lim_{n\to\infty} K_n \frac{(x)}{\sqrt{n}} = K(x)$$

where K(x) is independent of n.

From these computations the fixed maximum vertical deviation k between the theoretical and the empirical cumulative distribution function can be determined for a given sample size n and a confidence level of $(1 - \alpha) \times 100\%$, where

$$P[D \leq k] = (1 - \alpha)$$

If $|F_n(x) - F(x)| \leq k$ for all values of x, the hypothesis of F(x) being the underlying distribution function has to be accepted with $(1 - \alpha) \times 100\%$ confidence, otherwise the hypothesis must be rejected.

Characteristics:

Sample size: Ordinal or higher data: $N \geq 1$.

Considerations: Tests for continuous distributions.

Parameters of F(x) are assumed or known and form part of the null hypothesis.

Test Statistic: $D = \max_{-\infty<x<\infty} |F_n(x) - F(x)|$

Testing Process:

1. Hypothesis Statements

   Null $H_0: x \sim$ the hypothesized distribution with the desired parameter(s).

   Alternate $H_1: x \not\sim$ the hypothesized distribution with the desired parameter(s).

2. Statistical Test

   K-S one sample test

3. Significance Level

   Desired level of α and N (sample size)

4. Calculated Value

   Computed D statistic

5. Critical Test Value

   Enter the critical values of D in the K-S one sample test for the given α, N. If grouped data is used, use total N for critical values.

6. Decision

   If the calculated value of D is greater than the critical value of D, reject the null hypothesis at that level of α.

   If the null hypothesis is rejected, reject the hypothesized distribution and the inputed parameters.

CHI-SQUARE (χ^2) TEST

Probably the most widely used nonparametric test of significance is the Chi-square test. The goodness-of-fit technique is a test for significant differences between the observed distribution of data among categories and the expected distribution based upon the null hypothesis (5:371).

In the Chi-square one sample case, a null hypothesis is established from which is deduced the expected frequency

of objects in each category. Then, the deviations of the actual frequencies per category are compared with the hypothesized frequencies. The greater the difference between them, the less probable that the differences can be attributed to chance. The measure of the extent of this difference is the value of Chi-square. The divergence and the Chi-square value are directly proportional.

The formula for the χ^2 test statistic is

$$\chi^2 = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i}$$

where O_i = observed number of cases categorized in the i^{th} category.

E_i = expected number of cases categorized in the i^{th} category.

k = the number of categories.

There is a different distribution for χ^2 for each number of degrees of freedom. And depending upon the number of degrees of freedom (df), the size of each cell must be large enough to make the χ^2 test appropriate. If df = 1, each expected frequency must be at least 5 in size. If df > 1, 80% of the expected frequencies should be at least 5. Expected frequencies of one or more cells can be combined to meet this criteria, but with a loss of information contained in the data (9:382-3).

Characteristics:

Sample size: Large ($n \geq 35$); nominal or higher data.

Considerations: Tests for both continuous and discrete distributions.

Parameters may be given or estimated from the sample; if estimated, one df is lost per estimated parameter.

Test Statistic:

$$\chi^2 = \sum_{i=1}^{k} \frac{[O_i - E_i]^2}{E_i}$$

Testing Process:

1. Hypothesis Statement

Null	$H_o: x \sim$	the hypothesized distribution with the specified parameter(s)
Alternate	$H_1: x \not\sim$	the hypothesized distribution with the (desired) parameter(s).

2. Statistical Test

   Goodness-of-fit test.

3. Significance Level

   Desired level of α and df = N-1-(# of estimated parameters).

4. Calculated Value

   χ^2-statistic

5. Critical Test Value

   Obtained from χ^2 table with appropriate α and df.

6. Decision

If the calculated value of χ^2 is greater than the critical value of χ^2, reject the null hypothesis at that level of α.

If the null hypothesis cannot be rejected, sufficient evidence to reject the hypothesized underlying distribution is lacking (30:F-14).

COMPARISON OF THE K-S AND χ^2 GOODNESS-OF-FIT TESTS

> The Kolmogorov-Smirnov one-sample test treats individual observations separately and thus, unlike the χ^2 test for one sample, need not lose information through the combining of categories. When samples are small, and therefore adjacent categories must be combined before χ^2 may properly be computed, the χ^2 test is definitely less powerful than the K-S test. Moreover, for very small samples the χ^2 test is not applicable at all, but the K-S test is. These facts suggest that the K-S test may be in all cases more powerful than its alternative, the χ^2 test [27:51].

The assumption of continuity of the cumulative distribution is one that is never met in practice--it is only approximated. As a result, the K-S test tends to be conservative. In the practical research setting, the probability of rejecting a true null hypothesis is likely to be somewhat smaller than the level of significance specified by the researcher. This problem may be intensified by grouping the data (25:210).

> Ordinarily, when all of the assumptions are met and the statistician has a choice between two statistical tests, the more powerful of the two will be the one with

the most rigorous assumptions. Thus, for a given sample size, one would anticipate that the Kolmogorov statistic would be superior to the Chi-square when the data are continuously distributed. However, the Kolmogorov statistic suffers great loss of power if this assumption is violated, and this is quite likely to be the case in the practical research setting. Some experimentation on the part of the investigator may be necessary to determine which of the two tests is most appropriate for his data [25:212-3].

A satisfactory theory of 'best tests' has not yet been developed for nonparametric methods; therefore it is necessary to rely heavily on intuition, and attempt to show that the nonparametric test selected is superior to other available tests of this type for the problem being considered [15:327].

For this reason the research effort accepted as passing any distribution which passed either the K-S test or the χ^2 test or both.

APPENDIX D

STATISTICAL DISTRIBUTIONS
USED WITHIN THE THESIS

EXPONENTIAL DISTRIBUTION

The exponential distribution model is often used in reliability. "For a good many years, reliability analysis was almost wholly concerned with constant hazard rates [26:185]." A constant hazard rate implies an exponential density function and reliability function. One important aspect of an exponential density function is that it is both a special case of a gamma density function (26:54) and a special case of a Weibull density function (26:49).

Let the continuous random variable t be the time to failure or the time between failures. Then the resulting distribution

$$f(t) = \lambda \exp(-\lambda t), \quad t \geq 0$$

$$f(t) = 0 \qquad\qquad t < 0$$

is the exponential distribution, where $\lambda = H(t)$, the hazard function.

The expected value of the exponential is

$$E(t) = \int_0^\infty \lambda t \exp[-\lambda t]dt = \frac{1}{\lambda} = \Theta$$

and the variance of the exponential is

$$V(t) = E(t)^2 - [E(t)]^2 = \left(\frac{1}{\lambda}\right)^2 = \Theta^2$$

where Θ is the mean time to failure.

The reliability function for the exponential is

$$R(t) = \exp(- \lambda t).$$

The cumulative distribution function is

$$F(t) = 1 - \exp[- \lambda t].$$

The hazard rate for the exponential is

$$H(t) = \frac{f(t)}{R(t)} = \lambda \frac{\exp[- \lambda t]}{\exp[- \lambda t]} = \lambda$$

which implies that the exponential distribution only applies if the failure (hazard) rate remains constant with age; i.e., the failure probability in any time period remains constant throughout the unit's lifetime (26:185). In other words, the exponential distribution is applicable as a reliability model for failure times only if the failure rate is constant over time (31).

In considering the exponential distribution model for describing failures of units in a renewal environment, the "complete lack of memory" property characterizes the exponential distribution.

$$P\{X > r + s \mid X > r\} = P\{x > s\}$$

where r and s are any positive numbers (12:156).

> This means that $P\{x>s\}$ is independent of r. In other words, if a piece of equipment has not failed during r time units its conditional probability of serving r+s or more time units is independent of r

> and is equal to the probability of serving s or more time units. Stated differently, if time to failure of a piece of equipment follows the exponential distribution, then aging of the equipment is immaterial [12:157].

Θ, the MTTF, is also a popular measure of reliability.

> The MTBF has meaning only when one is discussing a renewal situation, where there is repair or replacement. . . . Unfortunately these two quantities [MTTF & MTBF] are sometimes wrongly thought of as equivalent, probably because for certain simple constant-hazard cases they are equal. In a single-parameter distribution, specification of the MTTF fixes the parameter [26:197].

In a multiple parameter distribution, such as the Weibull or the gamma, the MTTF places only one constraint on the model's parameters.

WEIBULL DISTRIBUTION

In reliability applications, the Weibull distribution is a general two-parameter distribution. In recent years many reliability applications have been found for this distribution. The density function for the Weibull is

$$f(t) = kt^{m} \exp\left[\frac{-kt^{(m+1)}}{(m+1)}\right]$$

where m is the shaping parameter and k is the scaling parameter.

The Weibull model may represent not only a constant hazard rate but also an increasing hazard rate and a type of decreasing hazard rate by appropriate selection of the model parameters (26:190).

The cumulative distribution function is

$$F(t) = 1 - \exp\left[\frac{-kt^{(m+1)}}{(m+1)}\right].$$

The hazard model is

$$H(t) = kt^{m} \qquad \text{for } m > -1.$$

The reliability function for the Weibull is

$$R(t) = \exp\left[\frac{-kt^{(m+1)}}{(m+1)}\right].$$

In the Weibull model, if m = 0, the resulting distribution is the exponential distribution. Therefore, the exponential distribution can be thought as a special case of the Weibull distribution (as previously stated) or, alternatively, the Weibull distribution can be thought of as a more general case of the exponential distribution.

The expected value of the Weibull distribution is

$$E(t) = \Gamma\ (1 + \frac{1}{m+1})\ (\frac{n+1}{k})^{\frac{1}{m+1}}$$

where Γ (gamma)(n) = $\int_{o}^{\infty} \exp(-t)(t)^{n-1}\, dt = (n-1)!$ and n is real and positive.

The variance of the Weibull distribution is

$$V(t) = \left[\frac{m+1}{k}\right]^{\frac{2}{m+1}} \left[\Gamma(1 + \frac{2}{m+1}) - (\Gamma(1 + \frac{1}{m+1}))^{2}\right].$$

The Weibull distribution has two drawbacks. First, the model is a two parameter model, which means that there exists increased difficulty in estimating the parameters. But with modern computer applicability and expertise, this drawback should be minor. The second drawback is within the model itself. The Weibull model cannot accurately represent the linearly decreasing hazard rate, which is useful in describing early failures. "However, with an appropriate choice of k and m one should be able to minimize this effect [26:190]."

GAMMA DISTRIBUTION

Another distribution used in reliability is the two-parameter gamma distribution. The gamma density function is

$$f(t) = \left[\beta^{\alpha+1}\,\Gamma(\alpha + 1)\right]^{-1} t^{\alpha} \exp\left[-\frac{t}{\beta}\right]$$

where $\alpha > 1$, $\beta > 0$, $0 \leq t \leq \infty$, $\Gamma(x) = (x - 1)!$ when x is an integer. α is the shaping factor or parameter and a change in α will change the shape of the curve. β is the scaling parameter and will change the vertical and horizontal scales (26:54).

The gamma tends to be the key model for a standby system.

If we let $t_1, t_2, \ldots, t_n$ be the failure times of the on line component and the n-1 standby components,

> then the failure time of the system $t_s = t_1 + t_2 \ldots + t_n$. The most common model to choose for each time-to-failure distribution is the exponential [26:54].

If $\alpha = 0$, the exponential distribution is obtained from the gamma distribution with $1/\beta = \lambda$. If the α parameter is defined to be strictly positive, the Erlang is obtained from the gamma model. Therefore, the gamma distribution is a general case of both the exponential distribution and the Erlang distribution.

The gamma cumulative distribution is

$$F(t) = 1 - \left[\sum_{k=0}^{\alpha} \frac{1}{k!} \left(\frac{t}{\beta}\right)^k \right] \exp - \frac{t}{\beta} \quad ,$$

where α is a positive integer.

The gamma reliability function is

$$R(t) = \left[\sum_{k=0}^{\alpha} \frac{1}{k!} \left(\frac{t}{\beta}\right)^k \right] \exp - \frac{t}{\beta} \quad ,$$

The expected value of the gamma is

$$E(t) = \beta(\alpha + 1).$$

The variance of the gamma distribution is

$$V(t) = E(t)[\beta] = \beta^2(\alpha + 1).$$

ERLANG DISTRIBUTION

The density function for the Erlang distribution is

$$f(t) = \frac{(\mu k)^k}{(k-1)!} t^{k-1} \exp[- k\mu t] \qquad \text{for } t \geq 0,$$

where μ and k are strictly positive parameters of the distribution and k is restricted to being integer (14:417). In reliability, μk is set equal to λ, where λ is strictly a positive integer and the Erlang density function is

$$f(t) = \frac{\lambda^k t^{k-1}}{(k-1)!} \exp[- \lambda t].$$

The Erlang distribution is a special case of the gamma distribution in which the k parameter is defined to be strictly positive integer. In addition, the Erlang distribution with k = 1; i.e., a first-order Erlang distribution, is equivalent to the exponential distribution (13:159).

The Erlang distribution is a very important distribution in reliability. Suppose that $t_1, t_2, \ldots, t_n$ are n independent variables with an identical exponential distribution whose mean is λ. Then their sum,

$$T = t_1 + t_2 + \ldots + t_n,$$

has an Erlang distribution with parameters k and λ/k (14:417). In reliability theory, the Erlang distribution

is used most extensively in studying standby systems. The above development (called a convolution) provides the reliability analyst with a means to assess the applicability of the exponential distribution. If the exponential distribution is convoluted over several renewals, the result is an Erlang distribution.

The expected value of the Erlang is

$$E(t) = t/\lambda$$

and the variance of the Erlang is

$$V(t) = t/\lambda^2.$$

NORMAL DISTRIBUTION

The normal, or Gaussian, distribution is a well known two-parameter distribution. Shooman showed that when a certain parameter which is a random variable is the sum of many other random variables, the parameter will have a normal distribution in most cases (26:50).

The density function for the normal distribution is

$$f(t_b) = \frac{1}{\sigma\sqrt{2\pi}} \exp - \frac{(t - \mu)^2}{2\sigma^2} \quad , -\infty < t < +\infty,$$

where the parameters are μ, the mean or mathematical expectation, and σ, the standard deviation (5:195). μ may be any constant and σ must be positive. The density function is a bell-shaped curve that is symmetric around μ (14:325).

The cumulative distribution function for the normal distribution is

$$F(t_b) = \int_{-\infty}^{b} \frac{1}{\sigma \sqrt{2\pi}} \exp\left[- \frac{(t - \mu)^2}{2\sigma^2}\right] dt.$$

The expected value of the normal distribution is

$$E(t) = \mu.$$

The variance of the normal distribution is

$$V(t) = \sigma^2.$$

The reliability of the normal distribution is given by

$$R(t_b) = 1 - F(t_b) = 1 - \int_{-\infty}^{b} \frac{1}{\sigma \sqrt{2\pi}} \exp\left[- \frac{(t - \mu)^2}{2\sigma^2}\right] dt.$$

LOGNORMAL DISTRIBUTION

A random variable t follows a lognormal distribution if its logarithm (ln t) is normally distributed with parameters μ and σ.

The density function for the lognormal distribution is

$$f(t) = \frac{1}{\sigma t \sqrt{2\pi}} \exp\left[- \frac{(\ln t - \mu)^2}{2\sigma^2}\right] \quad \sigma,\ t > 0.$$

The expected value of the lognormal distribution is

$$E(t) = \exp\left[\mu + \frac{\sigma^2}{2}\right]$$

The variance of the lognormal distribution is

$$V(t) = \exp[2(\mu + \sigma^2)] - \exp[2\mu + \sigma^2] \qquad (12:154\text{-}5).$$

The cumulative distribution function is given by

$$F(t) = P\{T \leq t\} = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{[(\ln t-\mu)/\sigma]} \exp\left[-\frac{t^2}{2}\right] dt,$$

which unfortunately does not have an easily integrated closed form.

The reliability function of this distribution is

$$R(t) = 1 - F(t).$$

BETA DISTRIBUTION

The beta distribution is a continuous distribution of a random variable with the density function given by

$$f(t) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} t^{(\alpha-1)} (1-t)^{(\beta-1)}, \; 0 \leq t \leq 1,$$

with parameters α and β. The density function is also often found in the form

$$f(t) = \frac{\Gamma(\alpha+\beta+2)}{\Gamma(\alpha+1)\Gamma(\beta+1)} t^{\alpha} (1-t)^{\beta}, \; 0 \leq t \leq 1.$$

The beta distribution in reliability finds usage in the maintainability area. Special cases of the beta include the uniform distribution, the student's t-distribution, and the F-distribution (14:324-5).

The cumulative distribution of the beta is

$$F(t) = \int_0^t \frac{\Gamma(\alpha+\beta+2)}{\Gamma(\alpha+1)\Gamma(\beta+1)} t^{\alpha}(1-t)^{\beta}\, dt, \; 0 \leq t \leq 1.$$

The expected value of the beta distribution is

$$E(t) = \frac{\alpha+1}{\alpha+\beta+2} .$$

The variance of the beta distribution is

$$V(t) = \frac{(\alpha+1)(\beta+1)}{\left[(\alpha+\beta+2)^2(\alpha+\beta+3)\right]} .$$

The reliability function of the beta distribution is

$$R(t) = 1 - \frac{\Gamma(\alpha+\beta+2)}{\Gamma(\alpha+1)\Gamma(\beta+1)} \int_0^t t^{\alpha}(1-t)^{\beta}\, dt, \; 0 \leq t \leq 1.$$

NEGATIVE BINOMIAL DISTRIBUTION

The negative binomial distribution is also called the Pascal distribution. The density function is given by

$$f(w) = \binom{w-1}{k-1} (1-p)^{w-k} p^k \qquad w = k, k+1, \ldots ,$$

where w is the number of trials considered and k is the

number of successes from w that is required in a given situation. The probability of success is p.

The expected value of the negative binomial is

$$E(w) = k/p.$$

The variance of the negative binomial is

$$V(w) = k(1-p)/p^2.$$

> The k^{th} success occurs on the w^{th} trial only if there are exactly k-1 successes in the preceding w-1 trials and a success occurs on the w^{th} trial. The probability of exactly k-1 successes in w-1 trials is given by the binomial distribution model . . . [20:112].*

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*The development of the Pearson XI distribution is beyond the scope of this research due to its mathematical complexity.

APPENDIX E

DATA FILES

GUIDE TO DATA FILES

Page

237	234	744	893	254	302	955	250	919	1399
1738	1359	772	1337	739	1539	1730	685	2393	1030
1475	929	1119	140	1467	1094	1208	1376	395	1177
1742	1953	2796	93	729	1065	1970	1124	993	1178
1075	1831	1349	2132	1369	1010	928	905	1271	1750
1384	2156	1115	1042	2254	1341	831	1191	1605	2546
1312	756	1498	1722	2382	1082	819	1277	1528	875
1431	355	1069	943	1122	1267	1508	387	696	787
1648	2051	657	1976	2095	310	1673	1304	576	773
702	518	1004	585	962	755	893	1010	878	0
455	790	997	472	1382	620	1099	1680	903	575
973	533	95	1122	1049	281	184	349	774	644
287	493	1179	323	388	825	460	1763	229	2161
263	859	209	2348	1062	439	927	741	1330	355
628	631	183	1136	772	601	555	595	705	531
472	554	303	231	308	419	269	297	257	195
215	250	245	282	367	141	366	420	452	680
1101	1362	416	1134	1662	946	2357	1672	1756	1550
1790	2112	259	227	860	869	137	663	174	134
407	136	207	442	482	261	793	585	208	583
1680	281	380	822	154	214	165	673	486	377
476	963	272	1090	461	661	468	351	1454	440
362	655	629	192	707	878	1114	247	939	1101
1428	872	313	164	1238	626	262	979	705	337
993	939	411	559	165	815	718	1464	472	381
489	221	1208	362	990	125	627	1466	237	380
157	635	1345	207	1280	616	989	345	466	813
865	1279	490	1090	307	862	1311	1051	1425	678
365	347	2611	3225	420	855	99	229	615	127
121	853	349	487	558	852	260	181	932	182
508	107	498	492	1089	1408	1099	828	164	383
414	417	103	334	109	153	163	93	483	414
451	95	2045	254	141	285	426	490	180	211
161	434	150	109	1001	183	217	701	240	164
101	565	120	222	122	299	212	377	129	

Convoluted Data for the KT-73 Unit

348	360	326	344	365	398	500	431	273	494
333	390	490	329	296	401	300	279	577	573
527	476	330	336	265	424	362	268	230	226
326	156	465	328	142	254	438	590	332	417
199	295	163	312	371	560	316	309	345	329
144	371	323	404	533	141	154	175	303	416
334	387	333	146	317	355	204	238	242	281
239	246	281	195	231	327	377	377	495	195
137	123	101	393	251	147	295	433	336	323
666	402	293							

Convoluted Data for the FLIP Unit

120	48	92	93	83	86	92	88	104	98
147	46	177	144	124	147	140	121	226	163
117	198	131	319	152	202	149	141	83	110
173	189	130	181	140	39	33	98	103	103
96	148	292	123	151	116	105	167	173	160
91	58	136	251	78	76	155	173	111	221
165	115	171	147	119	176	147	98	93	94
105	111	159	234	172	101	193	119	78	108
247	96	136	51	113	79	126	70	89	182
164	107	136	138	92	58	106	73	115	39
94	115	119	127	66	72	101	69	91	109
49	101	145	54	88	70	119	154	140	155
191	183	183	162	243	129	101	197	90	83
239	108	138	98	200	147	64	165	151	79
155	126	98	215	95	101	175	114	177	64
124	68	192	89	60	91	90	106	100	72
118	51	105	94	127	126	61	49	75	47
88	52	136	103	124	111	118	122	175	124
110	107	99	81	117	130	62	127	67	87
174	114	103	97	80	97	68	64	115	59
158	82	89	52	152	86	91	58	81	42
69	79	64	57	132	57	88	83	79	89
82	104	195	168	172	125	95	75	81	97
63	61	132	172	189	130	94	64	111	119
106	167	79	119	78	111	94	87	125	122
76	83	85	158	107	105	126	76	157	190
94	54	75	118	87	89	162	130	192	153
157	134	128	139	141	98	126	86	81	74
112	59	64	39	67	46	115	102	89	105
92	125	48	99	93	70	64	47	108	130
165	111	153	122	56	106	98	91	62	56
63	95								

Convoluted Data for the LN-15 Unit

237	234	744	893	254	302	955	250	919	1899
1738	1359	772	1337	789	1539	1730	685	2393	1030
1475	929	1119	140	1467	1094	1208	1376	895	1177
1742	1953	2796	93	729	1065	1970	1124	993	1178
1075	1831	1349	2132	1369	1010	928	905	1271	1750
1384	2156	1115	1042	2254	1341	831	1191	1605	2546
1312	756	1498	1722	2382	1082	819	1277	1528	875
1431	355	1069	943	1122	1267	1508	387	696	787
1648	2051	657	1976	2095	310	1673	1304	576	773
702	518	1004	585	962	755	898	1010	878	0
455	790	997	472	1382	620	1099	1680	903	575
973	533	95	1122	1049	281	184	349	774	644
287	498	1179	323	388	825	460	1763	229	2161
263	859	209	2348	1062	439	927	741	1330	355
628	631	183	1136	772	601	555	595	705	531
472	554	303	231	308	419	269	297	257	195
215	250	245	282	367	141	366	420	452	680
1101	1362	416	1134	1662	946	2357	1672	1756	1550
1790	2112	259	227	860	869	137	663	174	134
407	136	207	442	482	261	793	585	208	583
1630	281	380	822	154	214	165	673	486	377
476	963	272	1090	461	661	468	351	1454	440
362	655	629	192	707	878	1114	247	939	1101
1428	872	313	164	1238	626	262	979	705	337
993	939	411	559	165	815	718	1464	472	381
489	221	1208	362	990	125	627	1466	237	380
157	635	1345	207	1280	616	989	345	466	813
865	1279	490	1090	307	862	1311	1051	1425	678
365	347	2611	3225	420	855	99	229	615	127
121	853	349	487	558	852	260	181	932	182
508	107	498	492	1039	1408	1099	828	164	388
414	417	103	334	109	153	163	93	483	414
451	95	2045	254	141	285	426	490	180	211
161	434	150	109	1001	183	217	701	240	164
101	565	120	222	122	299	212	377	129	122
292	454	332	163	121	154	1676	1471	1715	1980
1681	1558	970	1776	1391	1292	1356	1655	2043	639
737	1140	598	919	2444	727	581	768	846	867
705	904	903	1319	742	1476	624	1203	1055	948
1669	628	497	956	729	801	473	894	807	1301
679	1155	776	914	930	1039	960	887	359	242

Failure Data for Cycle 1 of
the KT-73 Unit

325	471	732	672	1315	584	1753	686	897	555
791	1002	1373	631	1734	863	539	1171	984	1730
702	559	1474	624	591	462	592	849	1627	990
1577	1584	856	1470	973	857	629	580	1542	384
709	369	613	396	438	378	665	1021	1804	633
821	770	713	1653	916	554	858	524	1658	614
1831	655	362	834	438	843	465	304	1207	809
633	316	2263	723	1797	694	1538	1691	448	687
600	882	393	543	634	757	901	1263	556	353
919	1025	1165	1165	832	853	1344	675	548	1376
269	381	654	461	158	470	395	721	204	819
1236	433	1038	786	231	598	311	495	584	341
1184	442	843	623	314	271	1478	1127	744	1087
1310	315	1372	814	1327	1063	1100	659	664	272
951	510	334	815						

Failure Data for Cycle 1 of
the KT-73 Unit (continued)

60	9	25	570	220	748	14	694	32	202
215	17	1094	218	552	38	113	292	26	37
66	273	18	22	12	4	298	660	141	1379
91	648	217	12	101	261	55	1	270	607
15	426	305	1	227	422	73	967	148	14
360	172	50	236	8	262	187	516	1273	1
515	1	243	721	437	605	59	15	202	62
835	449	486	166	1212	176	132	395	396	1676
116	618	36	19	45	421	153	16	101	1
231	547	1	29	95	22	730	389	9	433
428	677	46	42	62	89	237	1	313	26
519	108	390	265	192	62	301	15	83	386
80	163	17	53	14	816	128	20	100	54
205	8	14	90	56	0	67	1	32	151
64	504	48	110	72	724	127	13	40	1016
447	186	297	417	1061	29	36	655	121	444
659	522	152	11	709	567	28	2	128	639
71	12	266	138	471	92	137	127	32	389
91	413	71	104	544	93	3	9	551	256
10	195	8	1	128	59	28	150	472	1
28	86	8	149	911	46	535	15	7	181
358	49	25	127	0	668	442	323	4	27
183	439	2	37	349	21	778	109	242	75
56	553	406	70	247	30	16	89	943	2
234	5	15	23	52	37	48	23	169	49
61	867	1910	508	32	4	771	96	997	316
26	41	504	896	60	32	114	132	508	63
5	34	7	213	17	95	18	61	76	142
332	111	50	725	517	239	2	359	101	7
57	2	294	74	46	4	132	400	160	5
46	23	72	127	494	452	1	133	299	1
104	112	1169	367	35	1096	17	631	274	211
244	1040	167	979	470	693	630	28	111	106
1	55	51	56	88	329	262	796	292	133
107	1	190	1327	19	4	352	890	110	68
91	472	212	87	2	9	4	101	516	877
559	16	22	119	319	2	563	197	12	465
429	463	1	392	358	191	405	192	64	3
1064	100	236	305	561	256	96	274	410	1218
284	56	29	173	732	85	52	1103	288	33
101	359	763	322	254	1	74	335	20	65
337	118	438	262	59	35	895	848	51	61
127	296	250	819	131	207	17	422	641	126
186	191	240	665	274	1658	73	163	749	213
5	77	6	53	385	284	670	219	446	304
4									

Failure Data for Cycle 2 of
the KT-73 Unit

105	91	89	1	0	159	10	4	76	116
176	101	185	655	13	227	221	674	94	259
31	103	131	76	53	301	89	804	46	304
18	9	37	838	2	528	333	3	32	66
38	163	170	351	1	963	96	123	316	374
34	705	513	28	511	296	895	32	1	135
8	24	17	331	92	449	30	7	65	1438
19	38	146	473	68	58	40	1424	1	188
117	18	181	25	63	233	58	1152	11	654
1	723	90	1	1	128	543	388	327	4
95	44	15	38	138	11	26	101	25	225
517	37	960	470	822	4	4	10	122	1
3	405	65	551	33	529	149	61	361	226
36	12	34	278	2	148	15	69	102	23
232	160	63	136	289	0	398	92	34	249
90	67	180	16	508	16	1	174	544	358
388	293	232	1588	11	9	672	253	880	118
14	159	18	312	112	84	38	681	91	12
170	540	323	8	454	125	469	578	4	373
11	573	241	200	918	31	643	36	2	244
901	86	231	908	246	15	674	201	147	593
7	106	186	61	11	89	12	509	46	30
388	119	288	119	49	213	71	1657	62	1211
113	169	103	790	32	382	4	1	188	563
287	462	43	524	1	672	44	34	2	479
22	24	105	410	1283	15	50	2	99	195
18	71	244	727	237	18	723	44	17	48
527	60	29	186	1	46	143	3	297	12
46	2	1	746	137	572	523	8	520	93
610	21	1	222	2	261	7	135	51	50
196	2	467	633	1	122	59	82	37	139
642	238	995	811	609	58	42	1	90	1
502	445	159	34	457	262	8	95	10	1
50	4	17	325	10	75	420	11	139	504
188	143	160	289	104	174	106	3	364	141
442	72	229	566						

Failure Data for Cycle 3
of the KT-73 Unit

35	442	55	1	5	32	51	20	188	61
59	305	32	487	75	12	78	5	137	13
219	39	38	58	766	723	1	1	11	116
17	261	667	54	278	195	200	6	2	42
43	109	9	109	897	365	90	26	1	263
523	135	1	42	6	196	48	153	48	61
499	42	1	78	161	151	68	123	47	131
70	31	22	40	41	142	199	31	294	34
149	265	2	99	51	31	540	273	108	472
93	2	118	18	129	0	138	838	561	140
133	84	6	14	300	69	10	50	58	8
90	16	2	1	57	628	111	26	59	1187
446	766	456	44	162	236	3	12	9	27
537	57	37	368	65	175	25	211	895	2
202	646	58	21	460	254	15	30	439	146
551	7	1	469	2	56	64	64	187	335
114	86	5	670	1	48	631	825	57	738
70	134	32	334	802	15	19	733	85	12
1	4	13	16	772	135	44	7	15	139
41	280	169	20	5	1	45	355	49	11
1	172	112	63	214	186	214	16	84	163
265	399	28	170	367	39	26	22	355	65
3	13	24	36	271	39	149	7	0	17
466	295	127	143	160	110	214	178	8	20
40	485	23	274	10	51	0	681	132	254
318	7	20	79	446	27	445	115		

Failure Data for Cycle 4
of the KT-73 Unit

147	6	295	2	677	33	85	8	446	107
87	7	1	246	452	1	840	7	106	216
87	47	36	296	378	118	24	49	131	22
5	31	118	631	112	630	250	4	101	73
163	173	3	156	28	190	218	162	1744	1
249	39	54	116	267	257	1	733	845	49
137	1	54	5	63	49	2	100	50	108
34	49	160	366	228	1	1	30	109	41
658	342	65	38	3	158	948	87	22	358
111	4	35	57	344	118	133	166	92	344
293	933	173	1	60	741	2	1	250	18
18	1	960	36	63	533	159	75	1	558
1	153	44	35	404	34	203	57	13	299
20	38	1	1216	367	5	52	78	2	73
11	370	215	9	86	1	571	327	6	165
335	873	33	293	11	1	711	172	89	62
38	1	26	126	19	156	20	423	64	356
170	83	46	219	49	497	28	3	306	561
157	55	83	19	57					

Failure Data for Cycle 5
of the FT-73 Unit

217	75	942	15	518	194	663	117	2	27
13	358	7	17	3	14	10	114	1	1
89	24	221	69	2	97	23	539	36	464
51	10	29	21	1	185	12	10	352	6
327	2	39	342	140	18	11	111	38	220
229	167	942	15	358	1145	38	598	7	40
1	16	20	12	10	11	34	6	126	65
110	15	316	2	220	229	2	1	112	151
63	270	1	92	21	14	11	265	64	418
65	10	36	288	6	580	672	52	27	1
159	13	68	109	2	9	93	12	76	92
38	370	318	79	11	1	2	21	67	9
-990									

Failure Data for Cycle 6
of the KT-73 Unit

894	1698	1775	1224	1545	2030	1725	2306	712	1182
1487	1142	1438	1637	1323	2731	645	1496	2892	1367
2332	1153	1397	1504	1373	2262	1596	946	720	934
1296	360	224	1198	118	779	1798	1933	560	1474
665	439	220	501	598	2044	243	389	1004	1146
1114	10	1665	1255	514	998	679	179	749	1160
723	534	2012	2363	453	595	1485	1632	911	770
951	1141	317	782	1197	682	554	1548	1103	2000
3411	132	270	132	220	333	1598	574	265	995
2399	1533	1892	672	1229	884	327	903	1189	802
442	1875	1609	360	1312	825	2071	35	784	188
284	239	165	65	152	122	32	22	113	139
88	61	47	132	136	1037	358	20	153	469
708	135	393	472	137	150	323	101	499	471
325	211	89	673	325					

Failure Data for Cycle 1
of the FLIP Unit

53	52	400	1325	306	395	521	373	409	1133
25	54	377	76	136	176	82	37	362	2228
479	422	604	385	148	167	13	427	133	183
1086	50	2603	159	2	12	20	5	35	20
5	76	120	321	0	87	396	506	88	49
400	773	643	452	36	622	78	47	256	1
306	3	47	50	411	337	355	4	180	607
13	65	136	358	14	210	78	46	277	62
296	20	3	20	61	193	10	337	150	10
2	156	37	1222	608	262	520	166	522	126
33	4	146	437	247	77	886	33	213	88
101	103	102	188	1	32	50	575	224	562
74	79	10	500	195	53	349	23	19	225
1	178	4015	8	15	5				

Failure Data for Cycle 2
of the FLIP Unit

335	107	303	30	401	380	1075	739	41	783
70	812	409	445	11	186	82	227	224	358
648	231	238	754	230	220	370	138	106	401
53	135	380	144	221	392	291	133	885	923
17	77	23	87	543	503	465	111	61	2
187	28	204	368	249	32	219	578	13	1146
18	1095	52	21	1265	50	36	212	158	36
319	531	440	153	67	21	370	56	1179	13
735	28	564	77	3	96	617	24	188	117
881	5	42	1354	1	19	368	550	243	676
155	820	273	143	195	142	78	27	21	233
530	850	526	8	13	68	18	386	49	347
1	129	114	213	271	98	437			

Failure Data for Cycle 3
of the FLIP Unit

161	82	174	34	150	364	406	208	197	111
223	186	348	156	624	389	111	18	87	479
177	7	221	103	129	15	93	324	18	362
25	234	221	613	21	157	771	345	349	136
104	290	12	192	143	516	106	100	8	470
29	94	183	37	139	485	167	165	35	5
906	8	233	108	194	113	31	172	27	132
189	1	225	265	57	124	178	89	61	523
13	324	34	267	130	66	414	941	462	274
786	129	398	76	152	352	663	548	113	117
1	165	4	92	227	418	2	843	224	1109
309	412	335	347	413	2	50	476	9	631
15	455	312	75						

Failure Data for Cycle 4
of the FLIP Unit

655	715	14	37	460	55	35	67	686	553
260	194	62	97	147	75	921	606	234	219
332	333	192	788	28	593	127	314	917	47
59	36	120	91	286	164	876	535	33	276
6	418	5	25	215	532	115	793	10	150
256	231	248	417	148	1052	2	22	88	180
130	408	155	166	211	1	461	68	72	57
23	263	9	25	15	14	132	423	42	6
30	250	167	60	95	1	269	4	233	243
23	680	414	121	646	66	577	207	227	585
1095	226	720	35	429	37	475	269	6	661
874	100	115	151	47	188	15	219		

Failure Data for Cycle 5
of the FLIP Unit

479	725	44	111	215	539	6	3	351	556
389	768	1539	1	14	94	376	5	24	83
13	425	128	66	53	311	343	6	31	3
438	432	210	588	247	724	20	1453	588	638
16	129	3	236	836	18	4	147	78	129
380	1	214	427	333	247	140	156	50	118
956	439	187	196	184	33	261	1	284	105
32	505	7	155	489	262	100	404	144	234
54	3	79	534	27	173	90	42	21	659
81	25	30	655	2	250	224	94	1	619
15	296	105	67	65	5	2	38	224	38
-990									

Failure Data for Cycle 6
of the FLIP Unit

703	80	1	34	5	10	431	188	5	26
127	19	12	307	59	11	199	44	788	510
393	35	153	38	39	85	413	27	146	81
130	121	343	150	4	7	188	1338	773	1
11	3	93	1007	430	754	485	216	1561	175
49	19	84	254	1069	90	66	17	5	49
8	121	72	279	212	12	263	1089	228	33
91	139	66	14	905	17	58	269	423	6
212	72	332	115	184	11	17	18	11	46
3	60	254	1069	225	471				

Failure Data for Cycle 7
of the FLIP Unit

29	94	8	462	337	69	499	80	306	30
115	378	519	163	517	240	51	309	290	135
436	1181	3	200	148	185	448	99	83	86
8	46	22	176	80	50	69	83	74	224
1055	257	333	658	350	240	448	169	543	34
32	52	0	1	1341	791	45	74	18	18
756	4	118	430	202	100	103	307	99	145
475	143	14	163	16	102	257	234	104	306
169	381								

Failure Data for Cycle 8
of the FLIP Unit

133	18	133	176	29	63	251	43	16	247
287	39	66	233	41	112	497	30	621	72
408	45	110	13	52	54	56	81	32	165
42	78	11	83	100	37	322	28	1	97
10	52	81	66	266	191	715	512	97	220
814	199	61	16	14	936	14	3	530	38
9	421								

Failure Data for Cycle 9
of the FLIP Unit

33	34	410	10	206	78	1	308	10	316
350	315	133	182	93	2	45	26	249	285
6	882	262	14	455	356	161	326	123	2
126	73	16	87	345	220	26	54	23	387
103	1214	745	33	284	721	191	151	4	

Failure Data for Cycle 10
of the FLIP Unit

771	185	512	212	501	572	609	210	308	607
1065	187	675	449	477	999	835	860	1364	1433
502	1437	797	1384	785	774	1042	938	416	803
1379	1146	867	1262	863	71	416	632	308	607
683	785	2475	826	1199	700	807	1151	862	471
700	393	1200	1714	424	387	723	902	914	1665
338	862	494	1028	696	1412	987	606	453	539
808	697	1275	1734	1251	647	1202	891	583	514
1693	476	1107	218	264	570	932	443	464	1045
747	773	598	710	605	283	617	278	837	200
535	502	689	804	360	325	140	285	426	124
186	146	592	122	243	164	339	513	773	982
1061	1619	1327	1187	1361	1893	592	778	1620	669
591	1512	636	754	371	1384	968	190	736	283
442	553	881	352	1360	374	356	1060	212	714
1076	360	917	411	1100	597	202	745	466	284
602	243	770	297	607	597	484	683	426	182
355	251	483	287	1148	393	983	836	446	768
755	642	363	513	448	249	292	839	437	421
422	328	1075	873	497	556	570	599	245	464
979	431	1054	526	581	355	1112	410	487	258
575	263	398	648	504	480	146	698	239	678
330	170	321	404	552	1093	513	839	394	564
295	575	242	270	239	873	739	557	1108	279
439	326	589	987	408	808	414	536	296	331
374	500	856	522	298	481	416	842	699	510
212	349	260	390	577	385	433	342	434	444
800	913	1689	891	748	1093	406	1001	418	715
843	544	319	414	690	154	256	130	287	187
629	681	696	652	337	774	196	197	378	200
279	184	605	399	681	778	657	555	339	726
175	546	213	174	156	495				

Failure Data for Cycle 1
of the LN-15 Unit

80	6	71	464	166	23	147	168	251	216
123	94	937	588	579	140	181	11	546	40
124	216	227	809	501	679	245	150	260	43
146	385	218	377	312	623	164	184	62	312
117	1	288	191	50	256	50	180	744	811
43	1	7	638	71	225	652	667	38	347
653	79	716	234	300	104	208	56	288	134
83	262	66	92	199	115	364	55	40	226
419	321	90	143	339	3	156	94	227	627
656	143	605	513	69	87	283	101	157	24
231	296	48	151	94	31	113	172	192	686
57	582	632	237	315	286	615	164	417	135
147	42	343	384	101	26	219	83	71	51
78	468	265	299	218	270	158	304	372	902
63	2	145	60	595	260	500	433	724	699
23	112	96	77	526	122	232	6	275	83
236	263	142	5	244	4	616	255	18	124
238	65	120	84	40	476	102	74	82	241
498	388	544	311	265	246	686	189	32	699
3	24	396	23	167	167	73	29	76	11
1	13	0	80	141	4	212	302	190	169
76	1	48							

Failure Data for Cycle 2
of the LN-15 Unit

203	136	191	103	13	113	11	357	332	4
129	26	11	256	32	182	239	193	197	0
397	176	133	849	83	421	50	170	5	106
52	212	67	18	75	47	146	17	15	7
7	546	9	64	115	55	47	192	20	172
17	33	5	7	131	2	21	11	9	48
503	59	350	59	40	96	123	167	43	118
4	5	95	369	116	96	215	96	12	195
213	15	16	1	375	68	22	14	45	2
85	0	5	11	99	59	9	200	4	12
26	203	307	170	52	211	612	85	141	131
101	128	75	29	172	96	84			

Failure Data for Cycle 3
of the LN-15 Unit

154	7	137	0	0	91	79	19	100	19
4	79	26	10	111	178	10	1158	8	22
13	6	9	19	1	192	109	8	0	90
5	4	27	12	39	184	50	27	127	79
27	124	27	12	27	8	1	28	70	

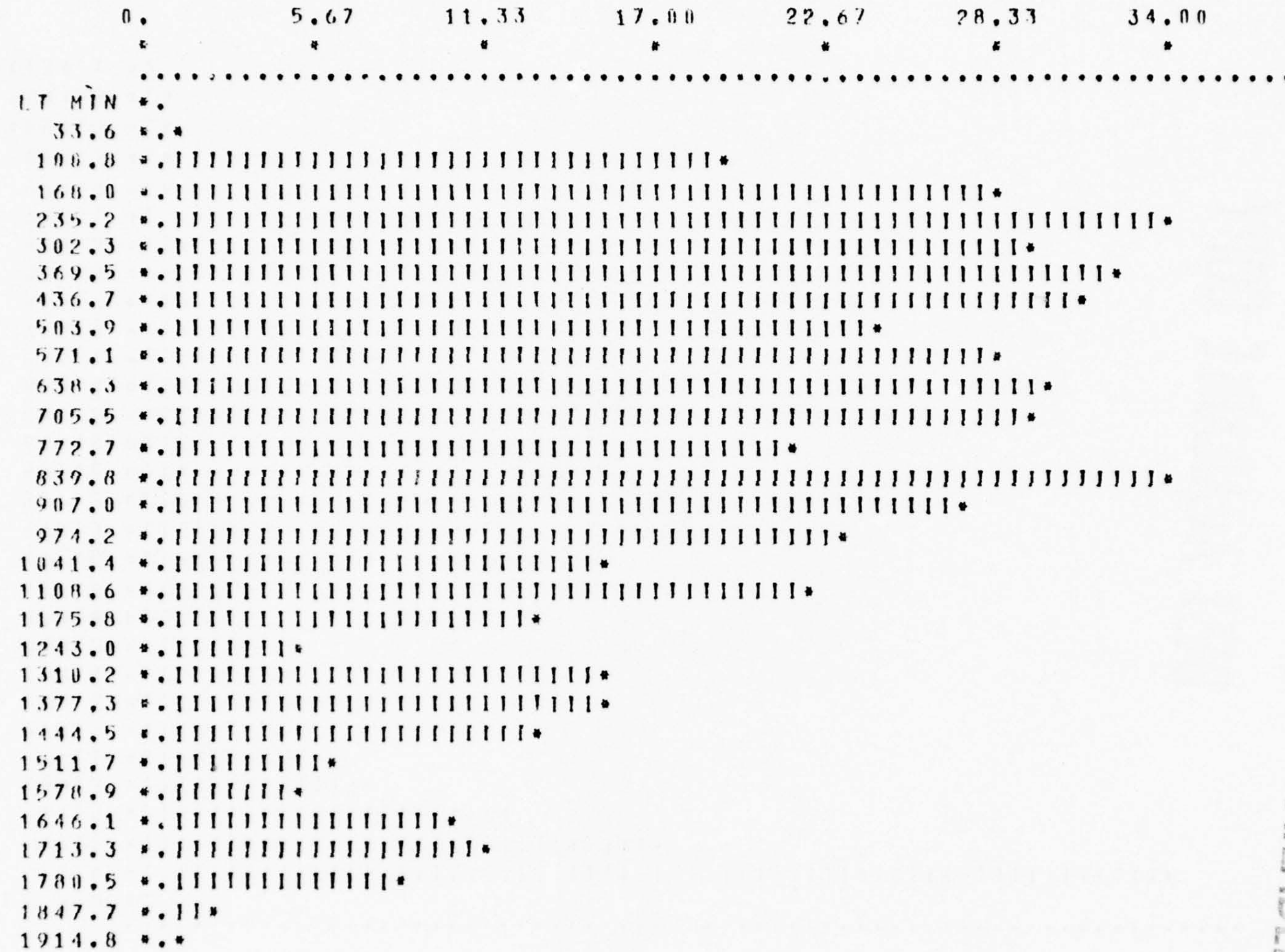
Failure Data for Cycle 4
of the LN-15 Unit

APPENDIX F

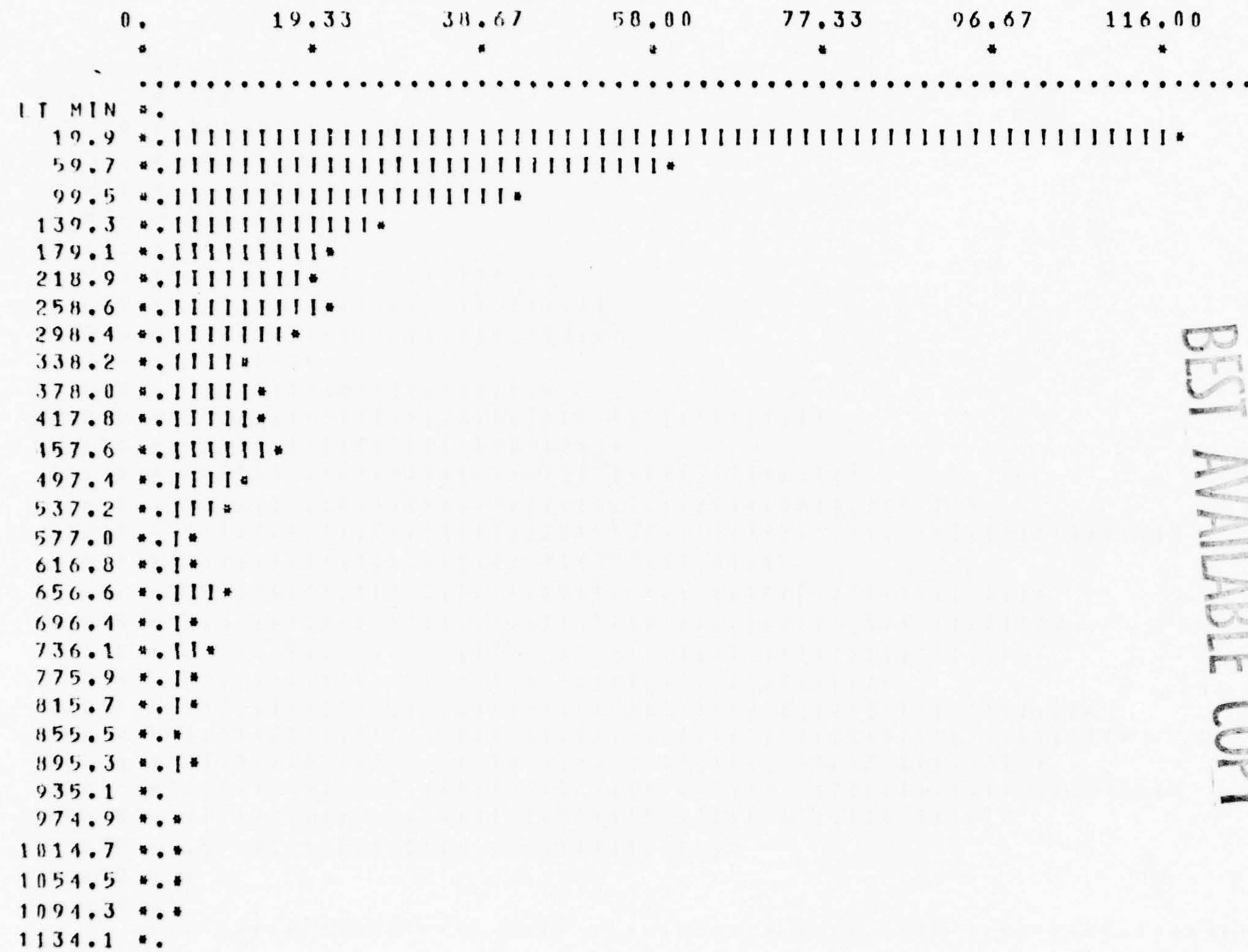
HISTOGRAMS

GUIDE TO HISTOGRAMS

Page



Histogram for KT-73 Cycle 1



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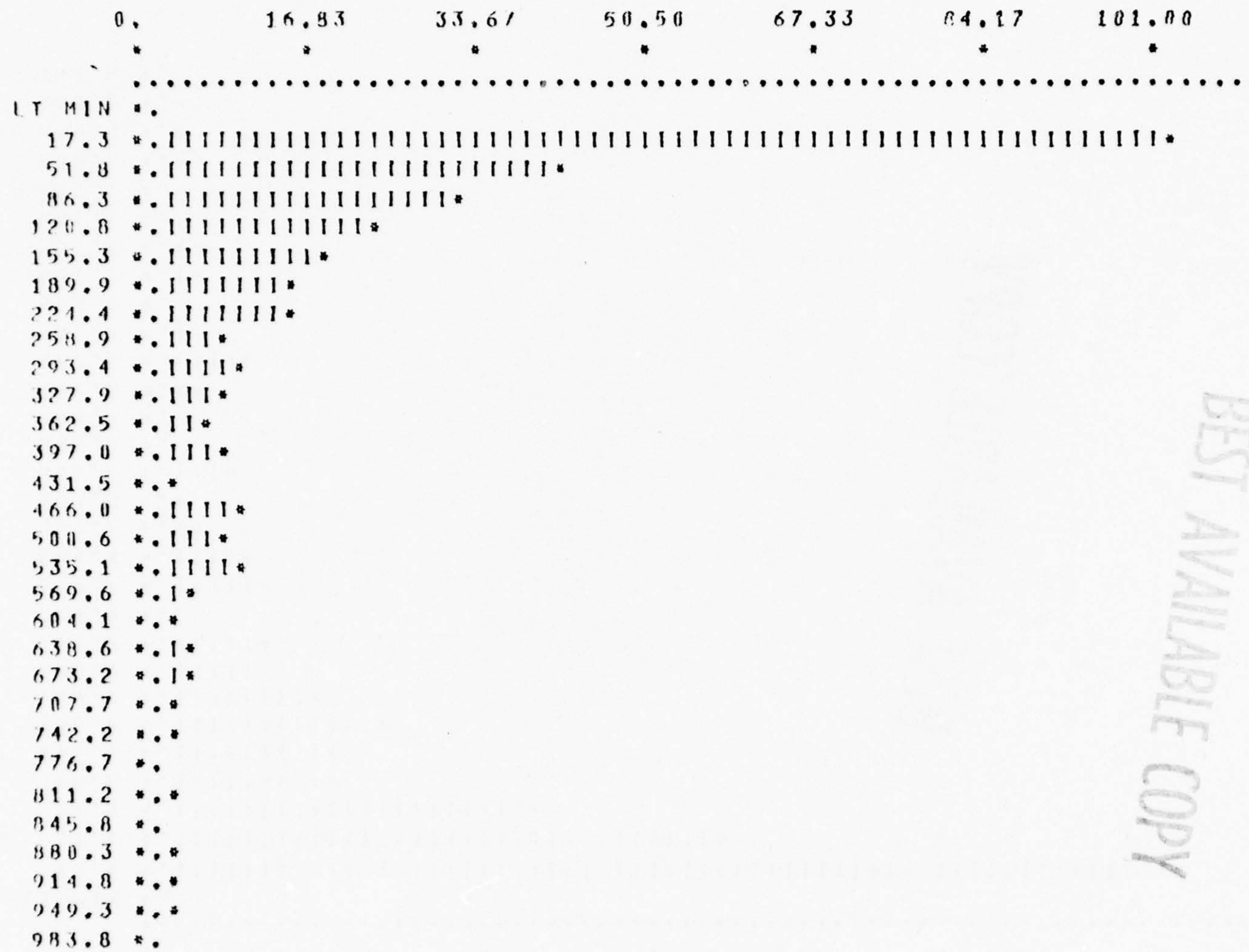
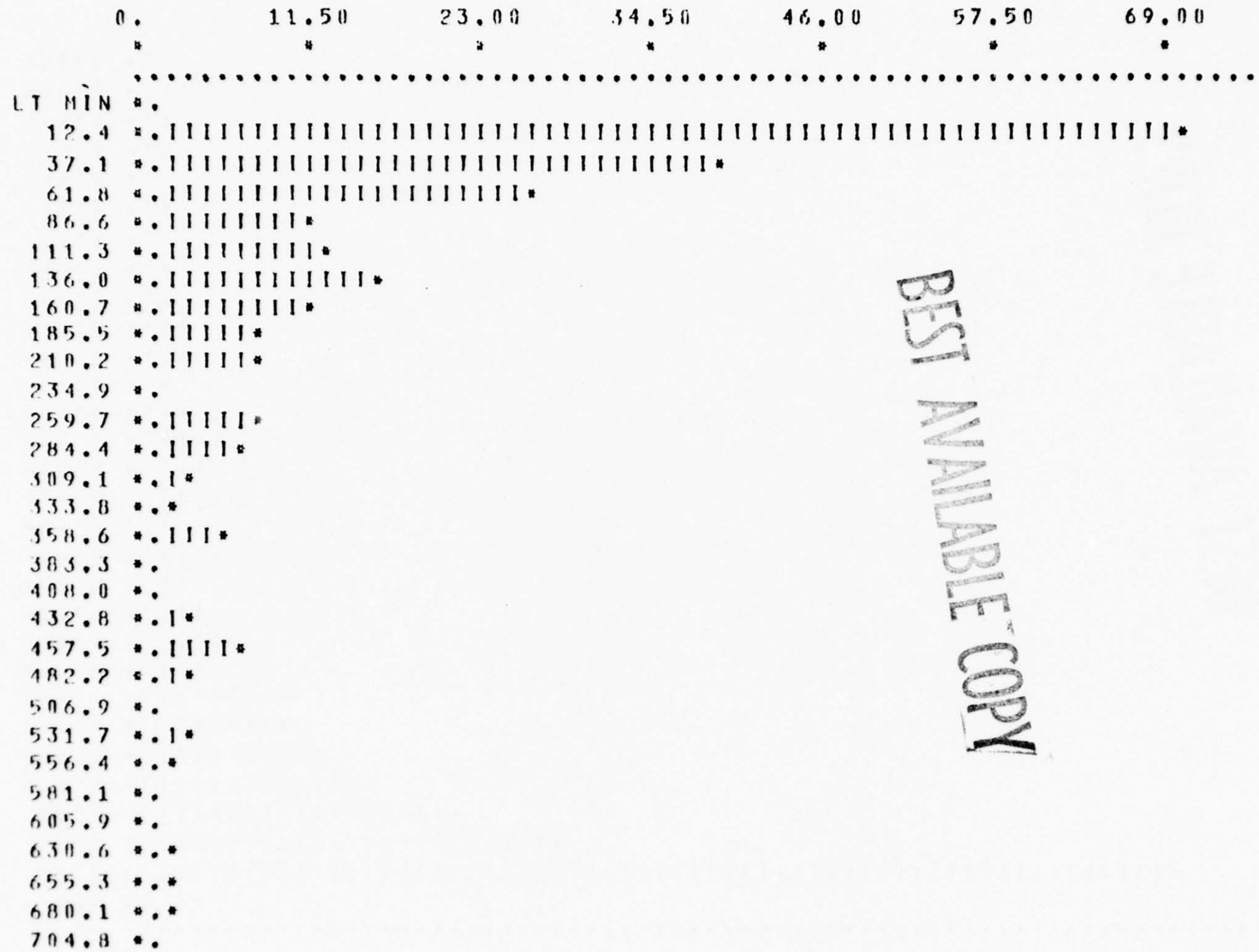
Histogram for KT-73 Cycle 2



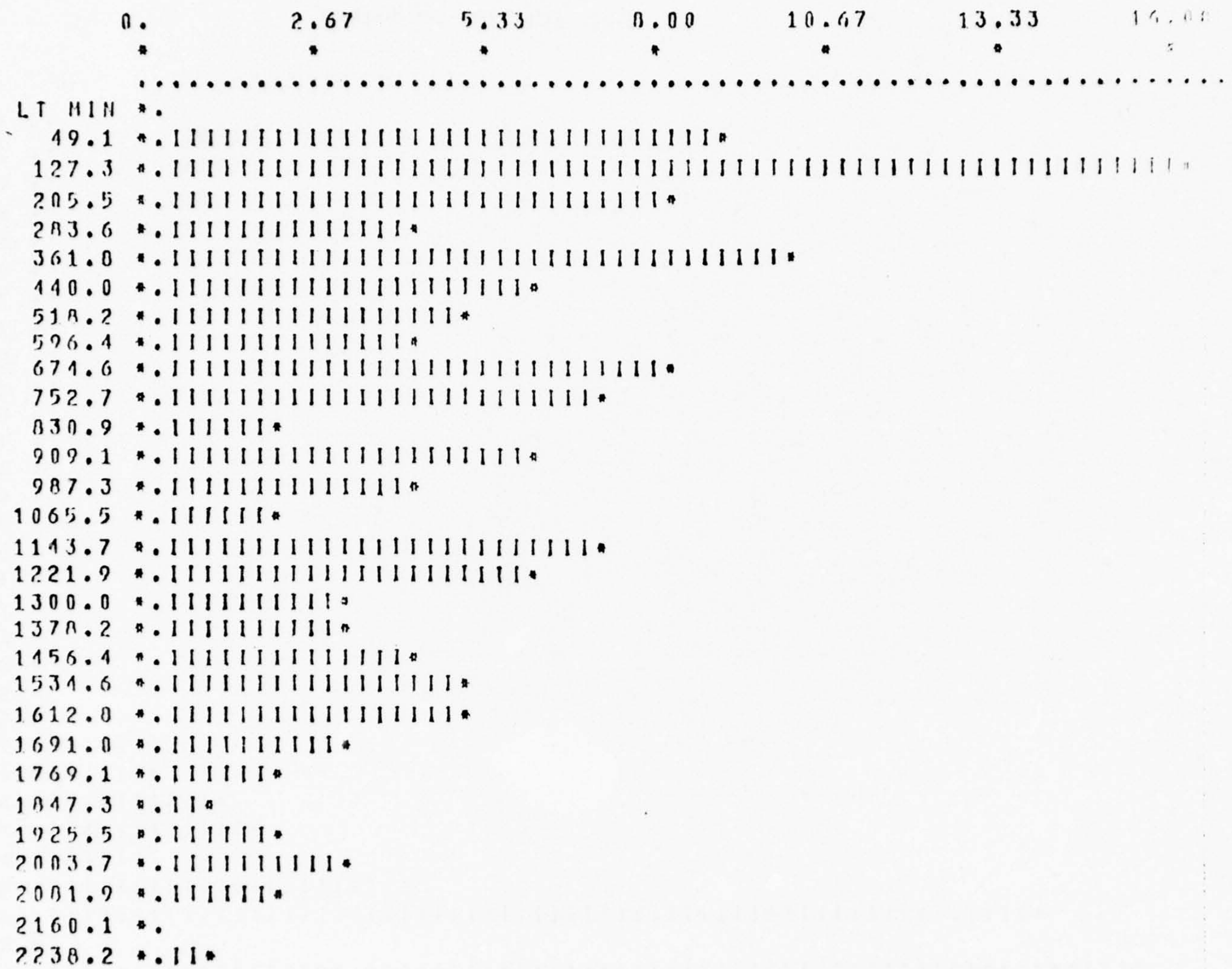
Histogram for KT-73 Cycle 3



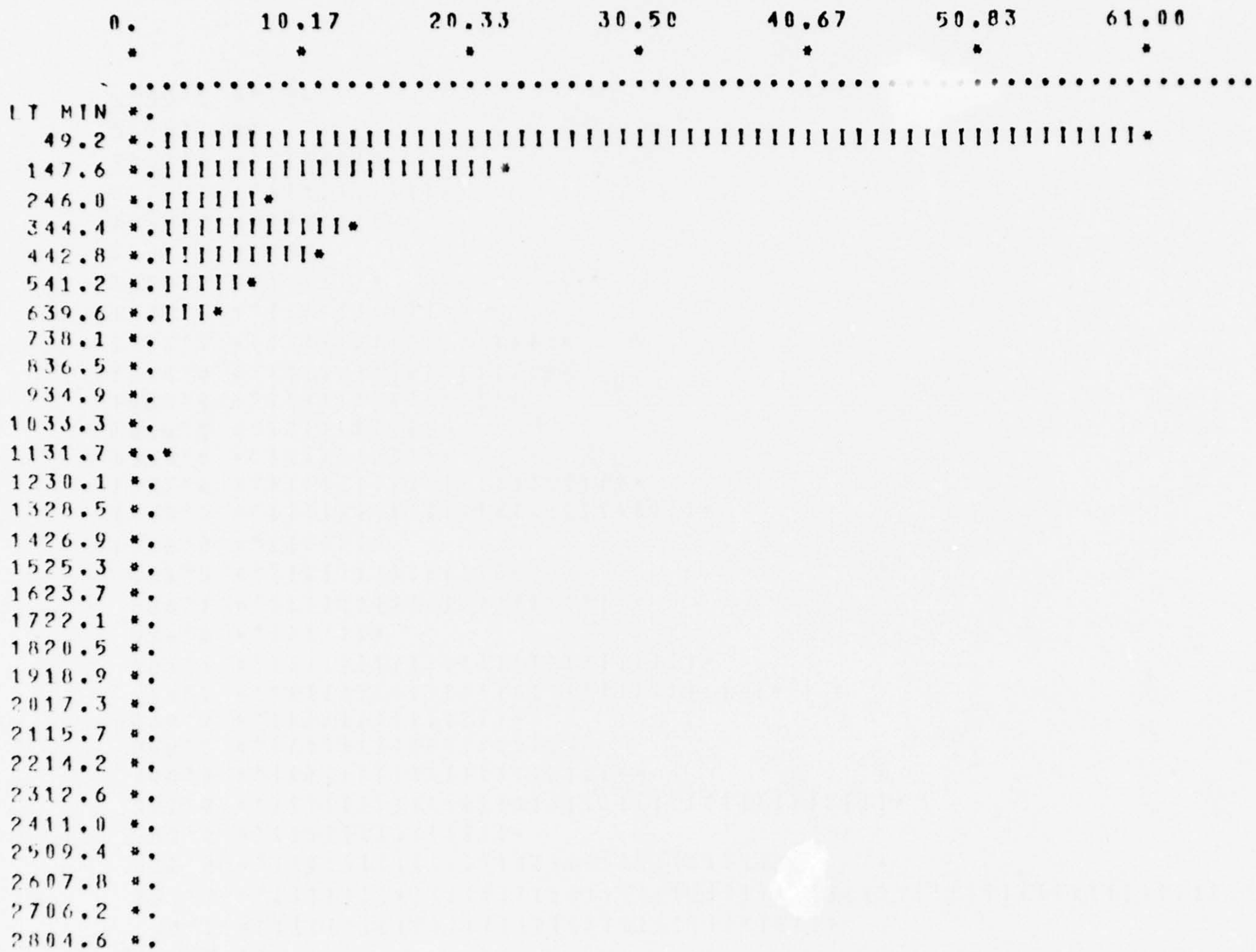
Histogram for KT-73 Cycle 4

Histogram for KT-73 Cycle 5

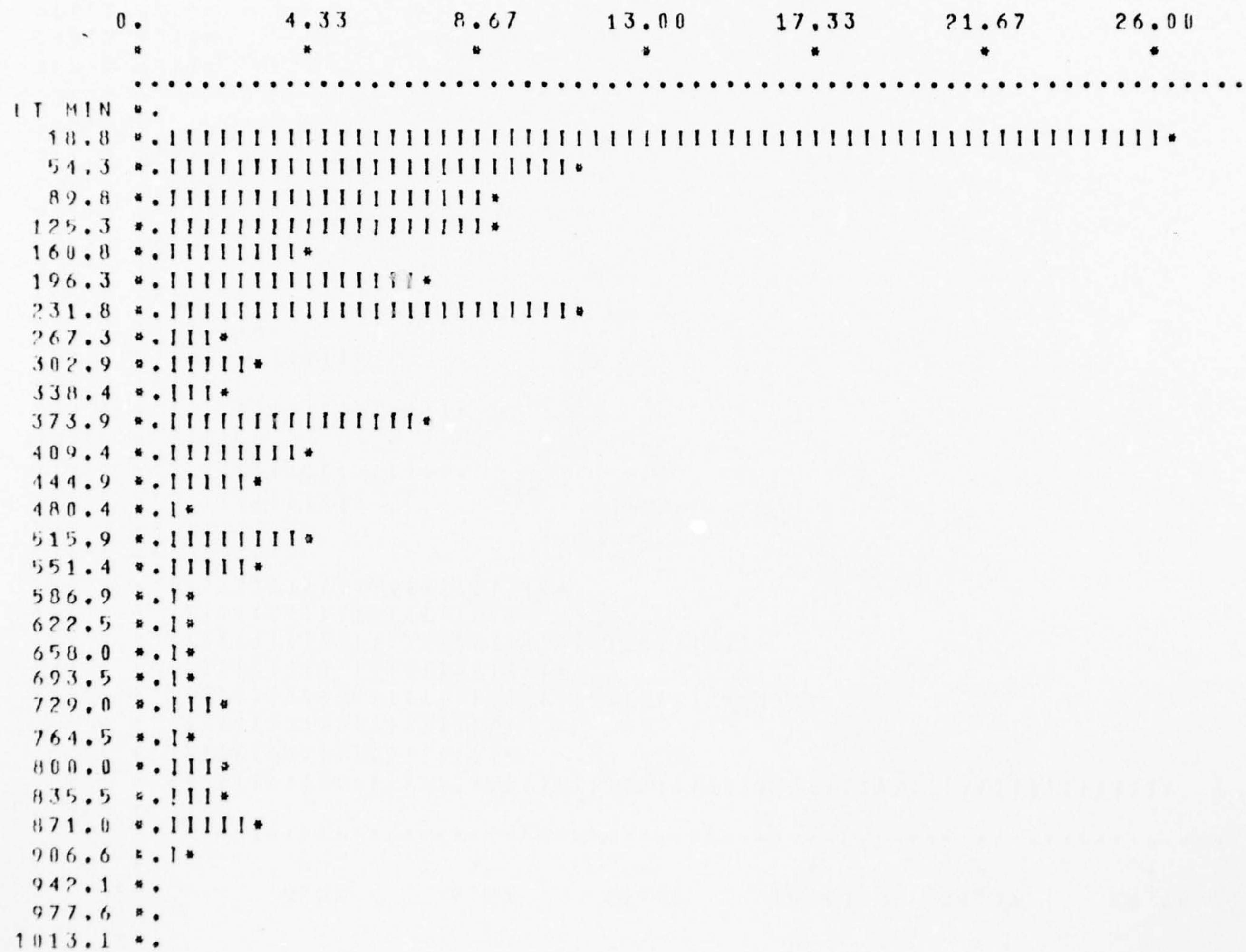
Histogram for KT-73 Cycle 6



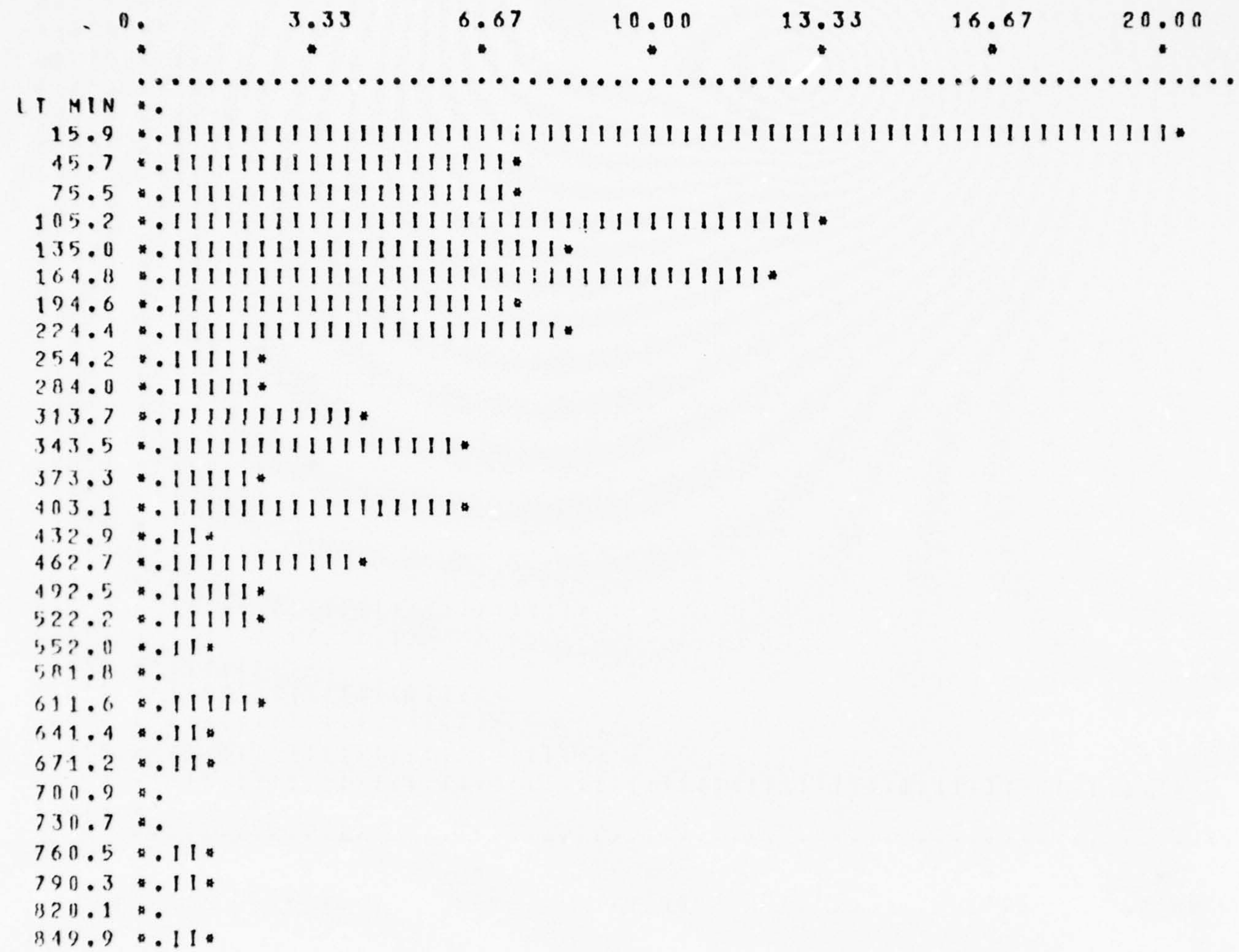
Histogram for FLIP Cycle 1



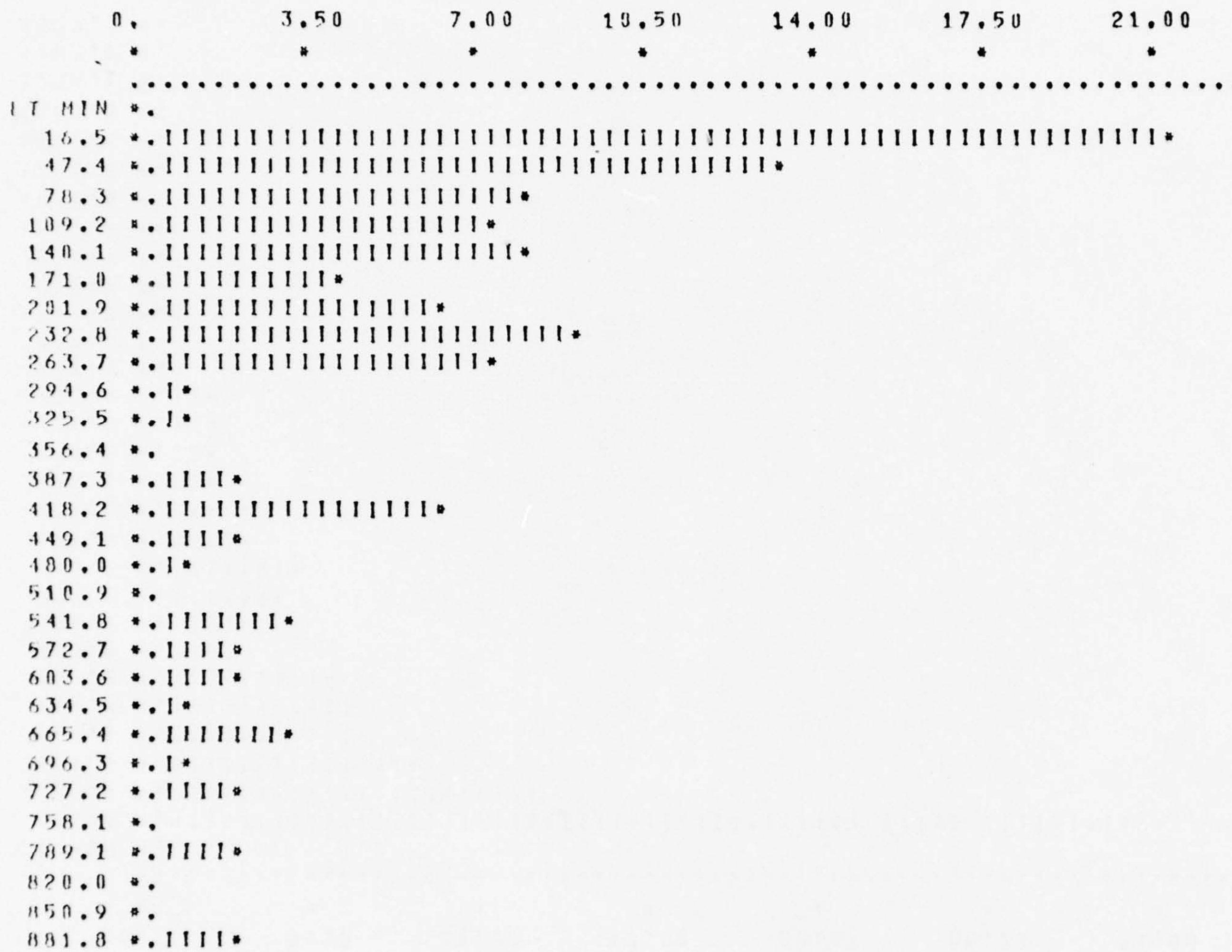
Histogram for FLIP Cycle 2



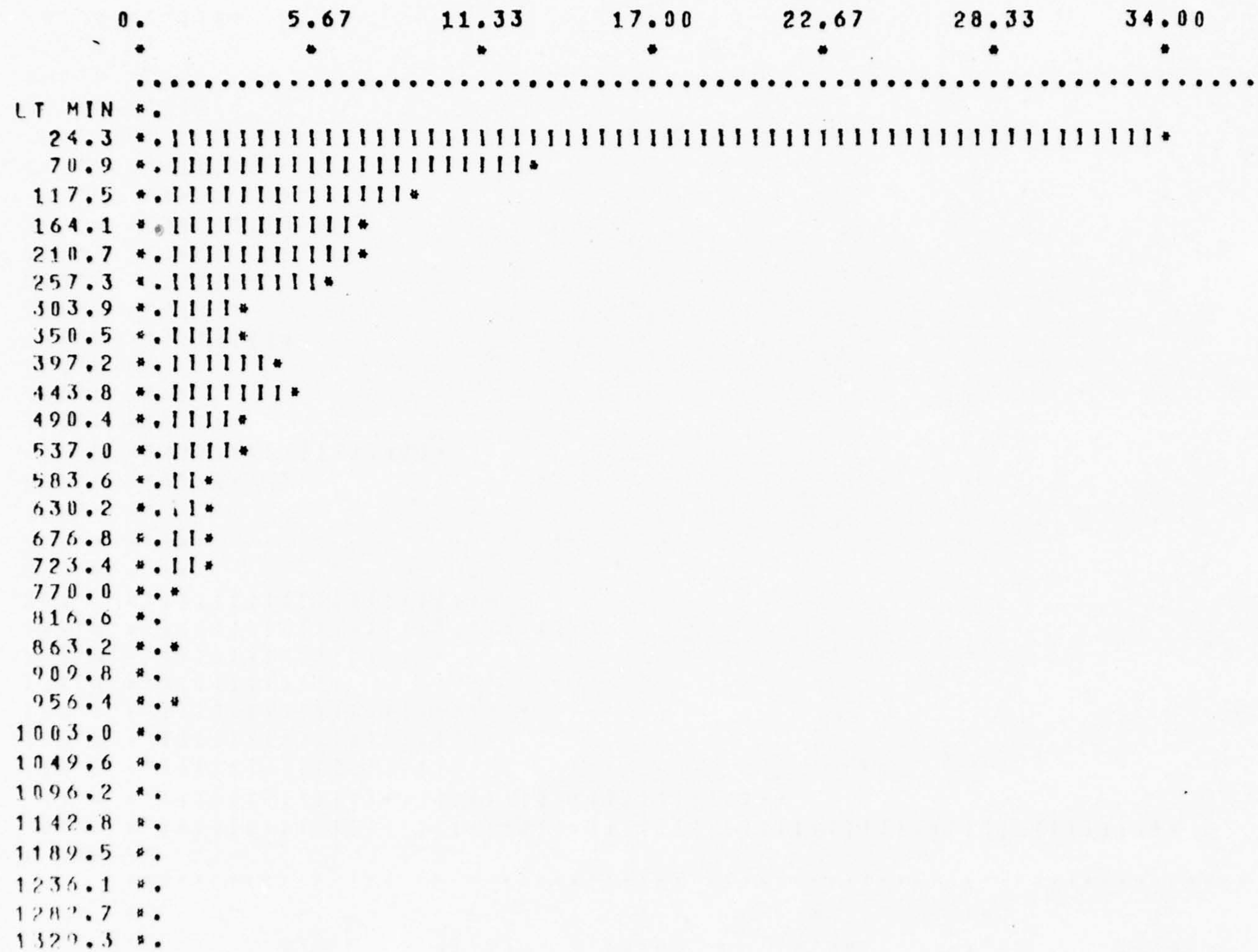
Histogram for FLIP Cycle 3



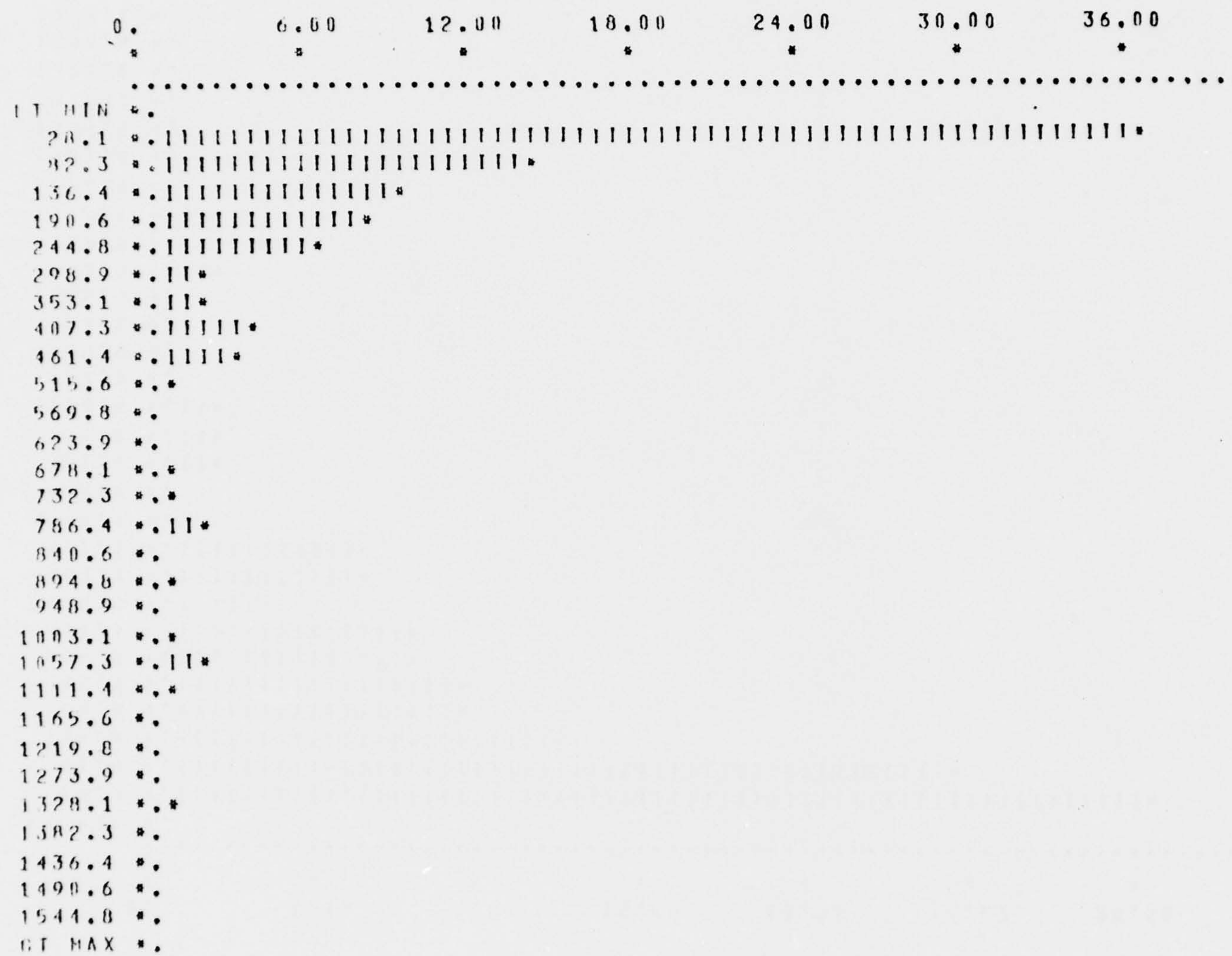
Histogram for FLIP Cycle 4



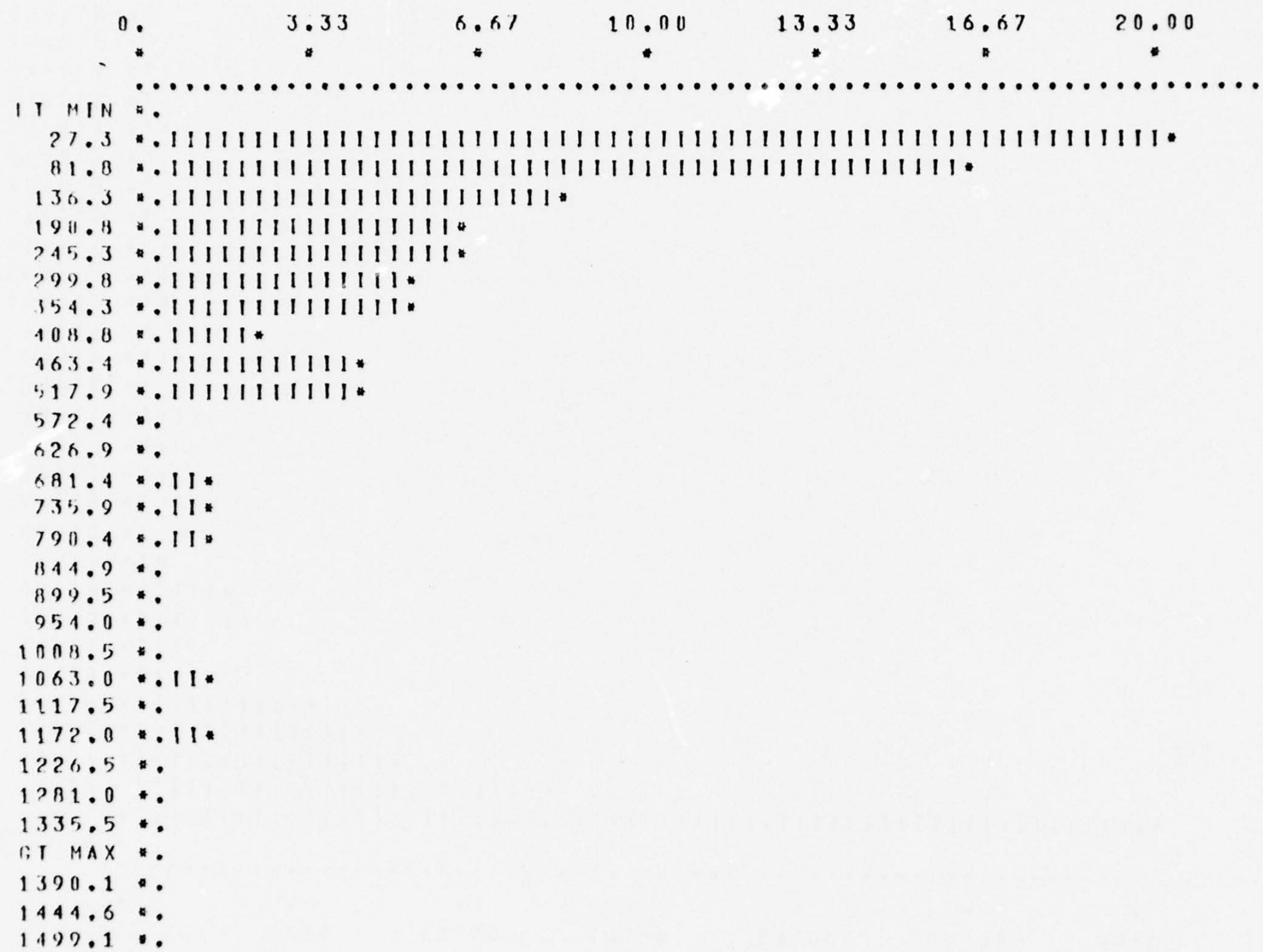
Histogram for FLIP Cycle 5



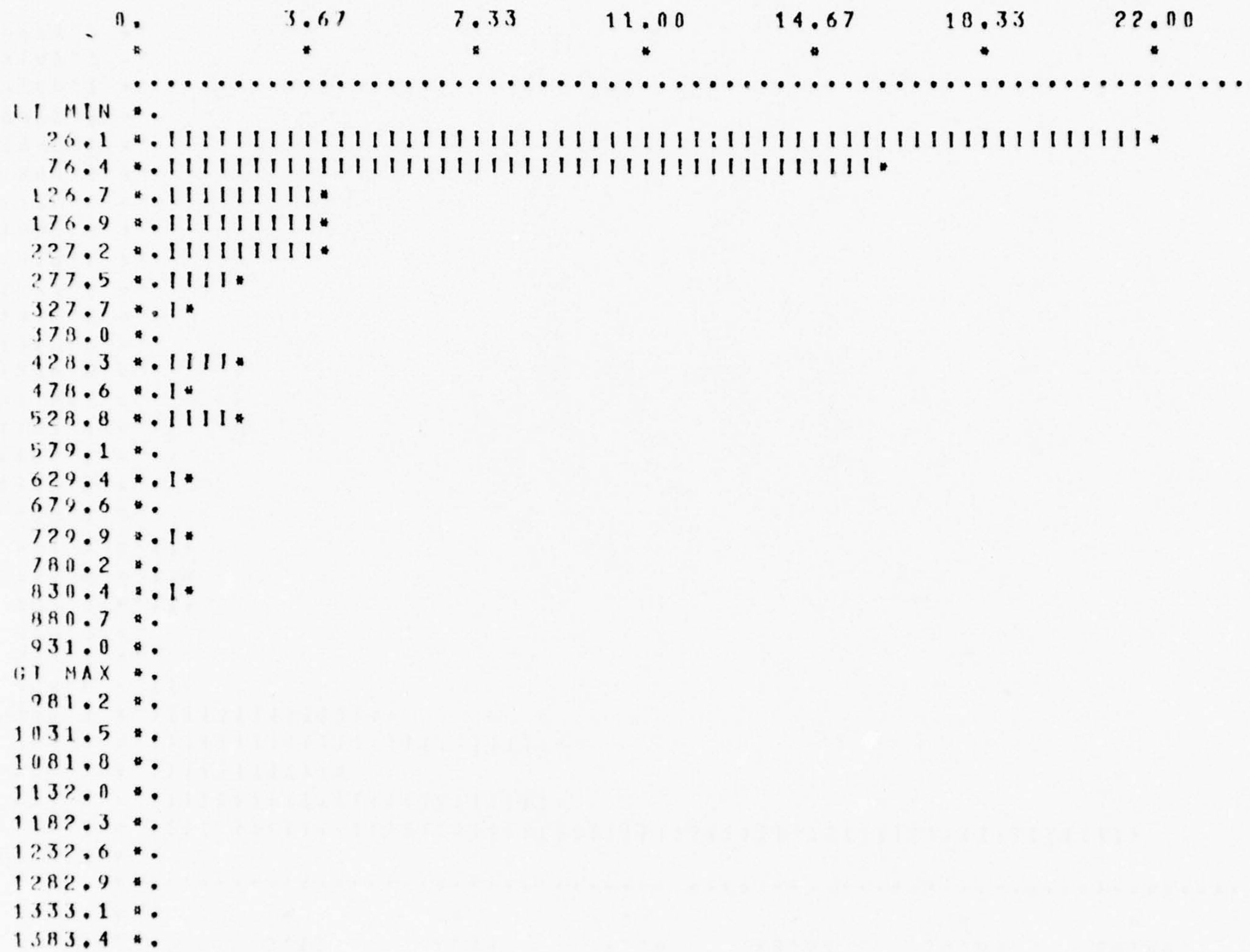
Histogram for FLIP Cycle 6



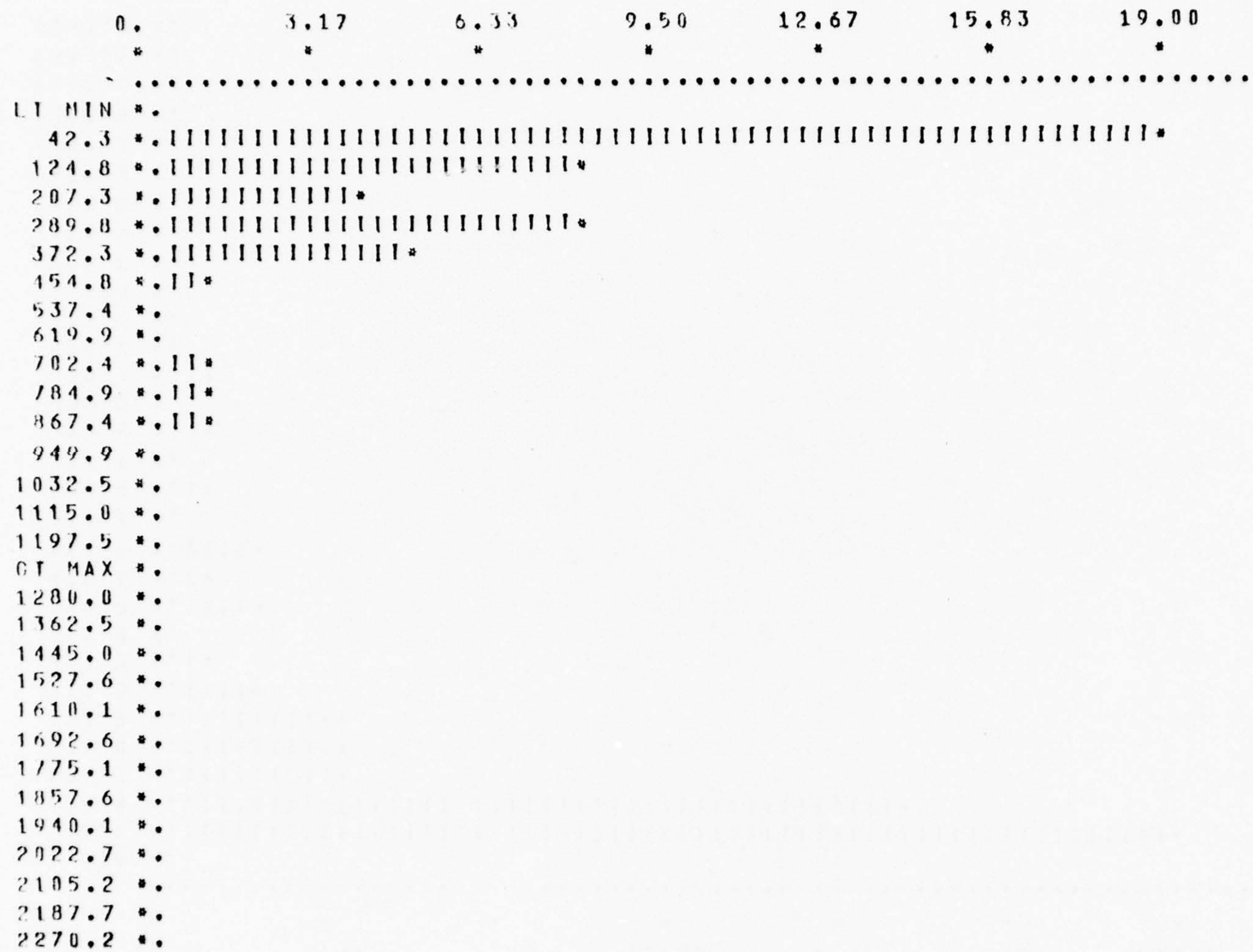
Histogram for FLIP Cycle 7



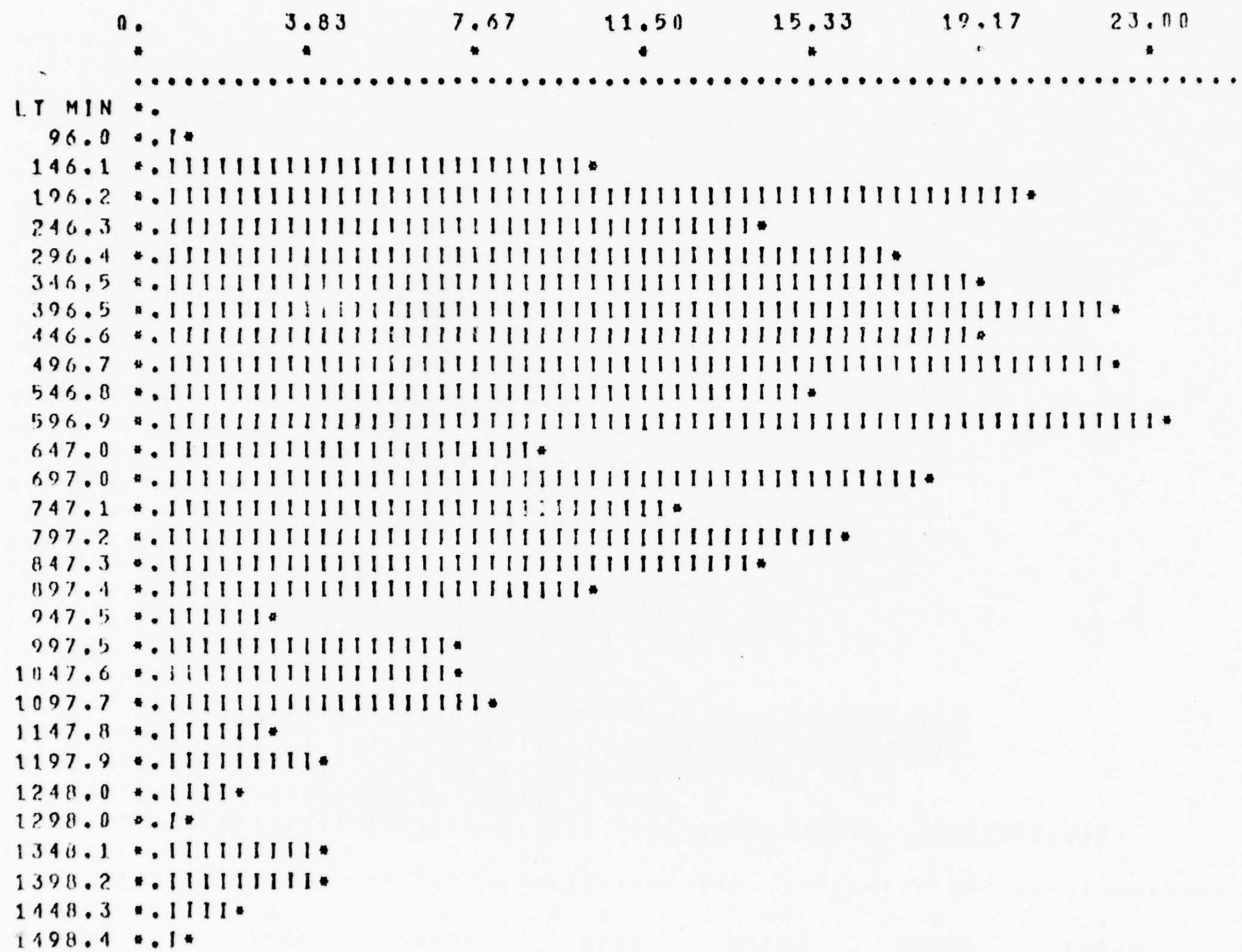
Histogram for FLIP Cycle 8



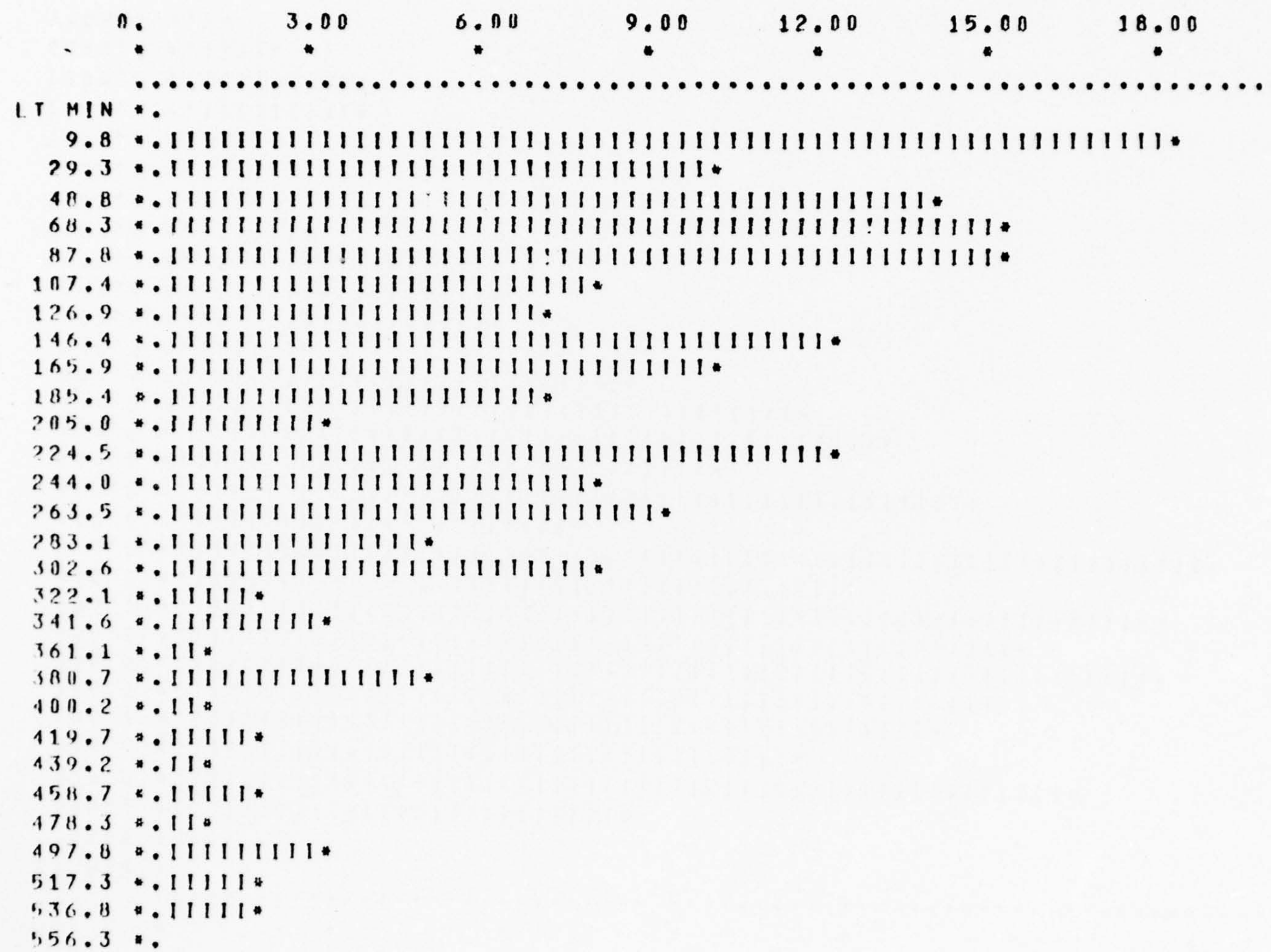
Histogram for FLIP Cycle 9



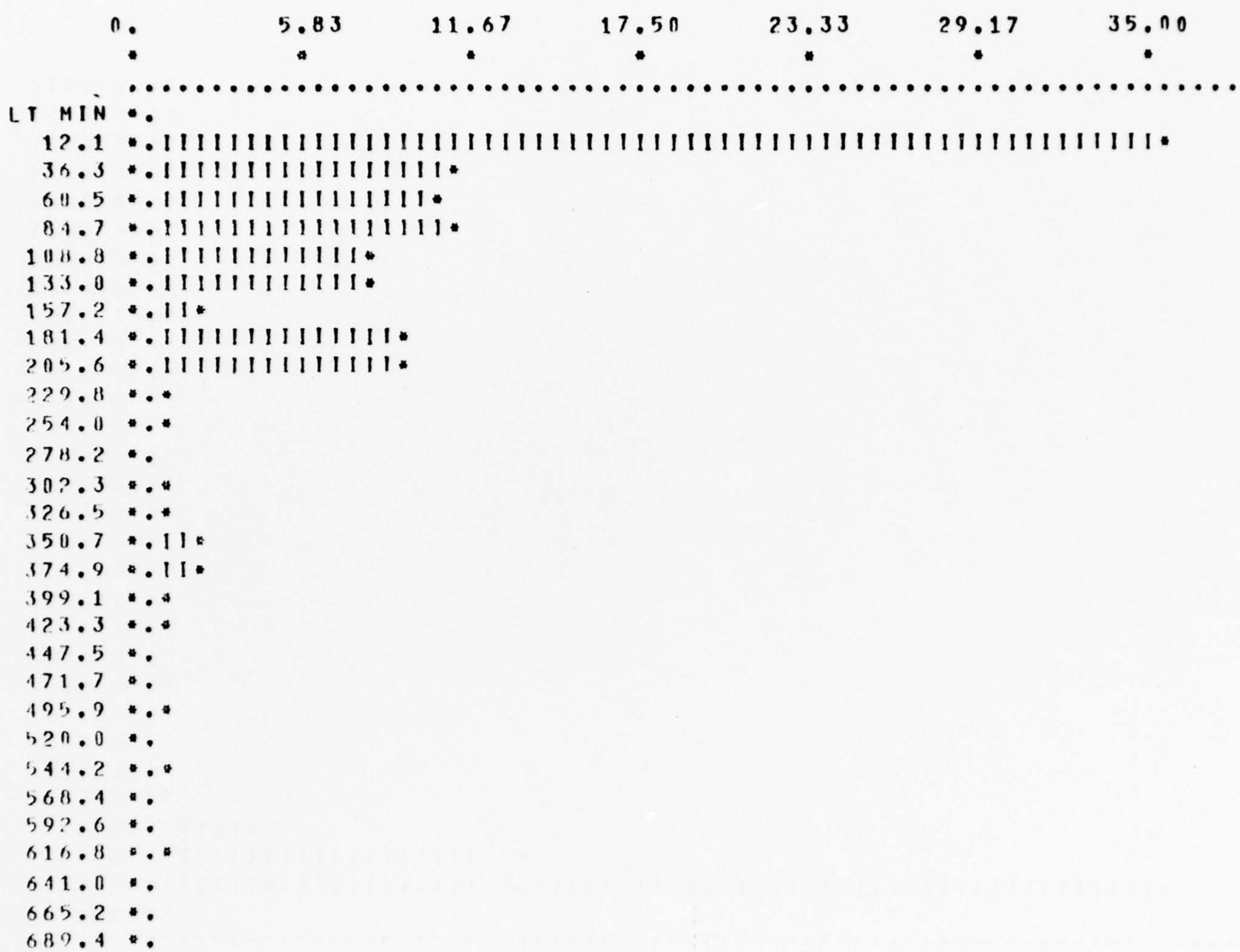
Histogram for FLIP Cycle 10



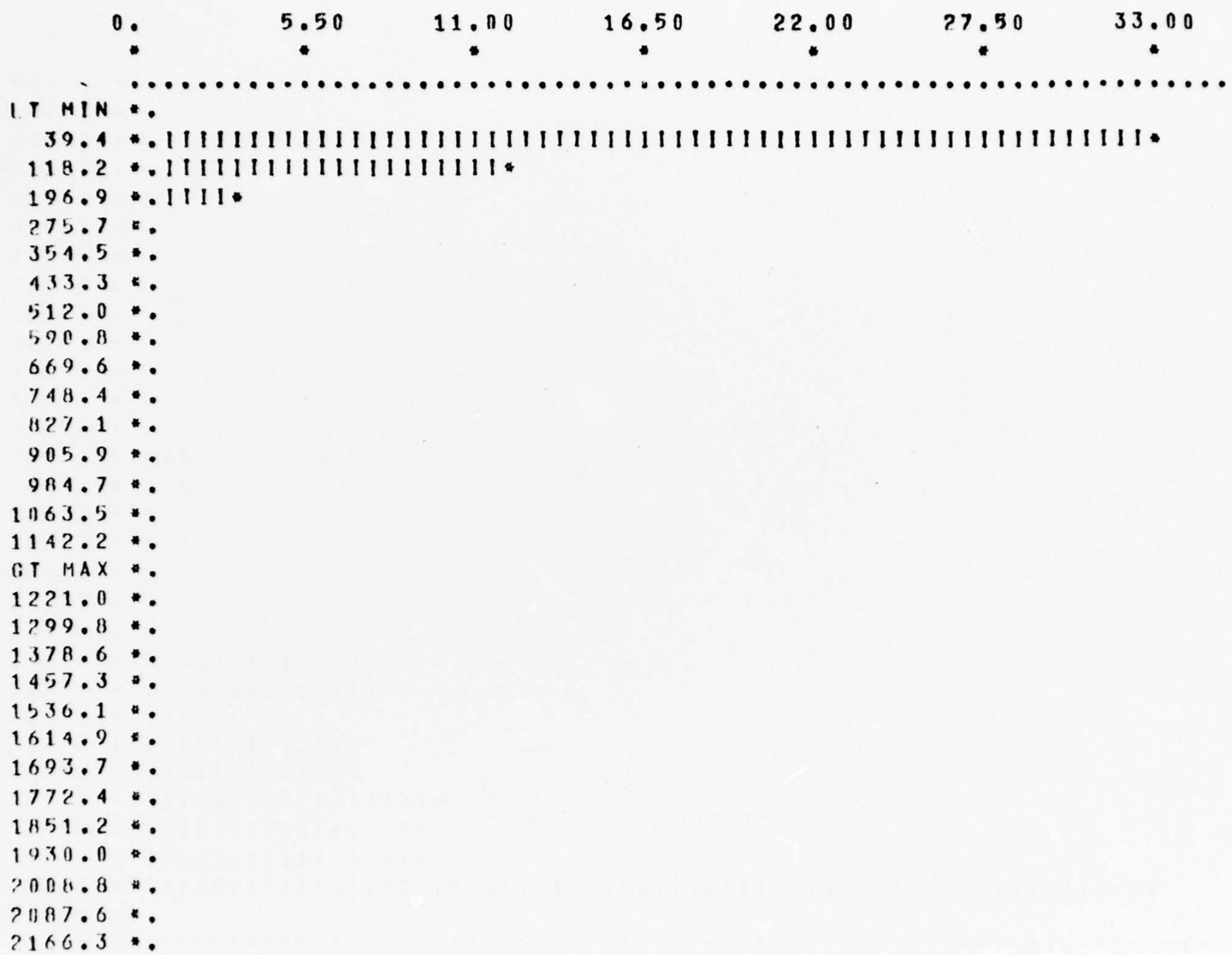
Histogram for LN-15 Cycle 1



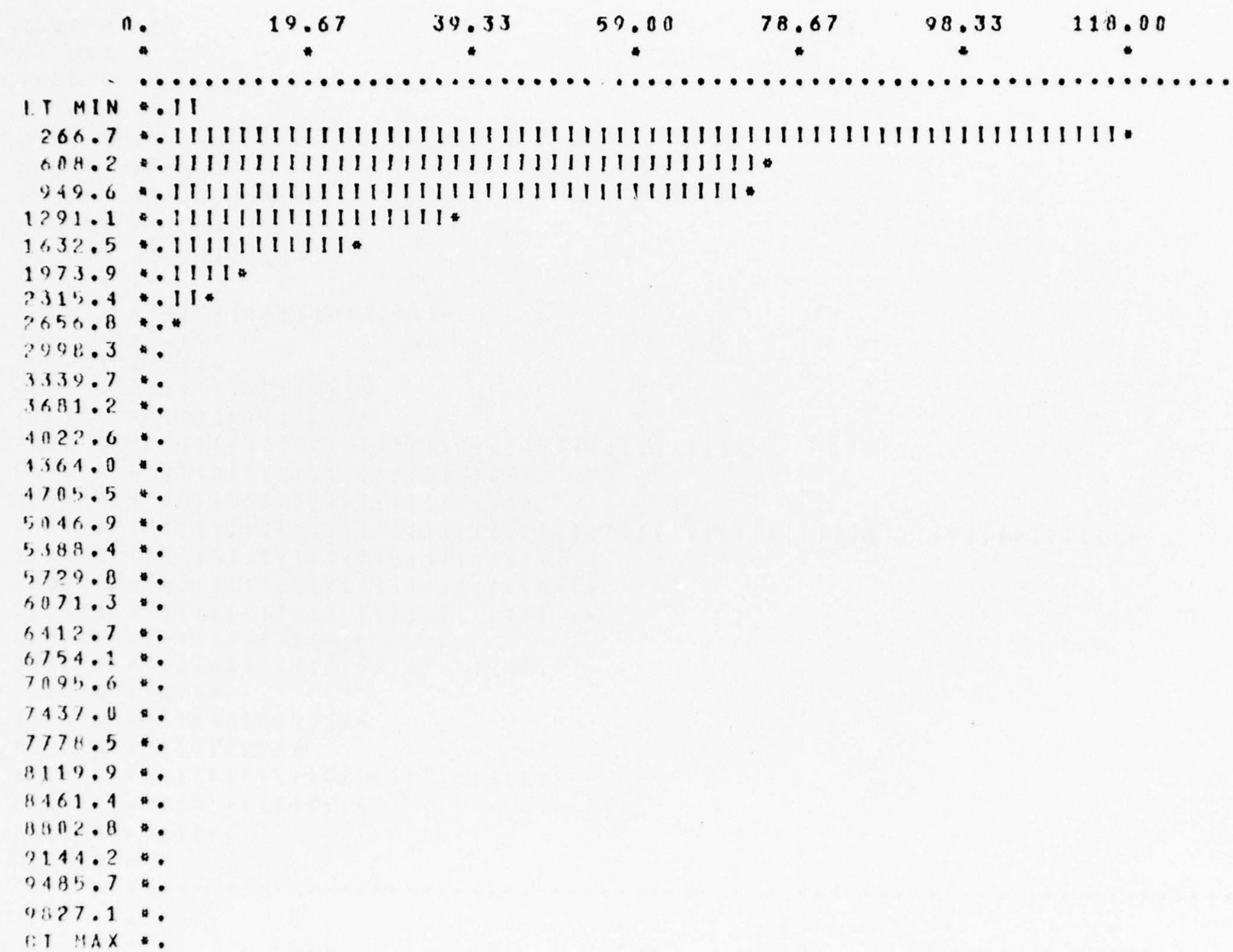
Histogram for LN-15 Cycle 2



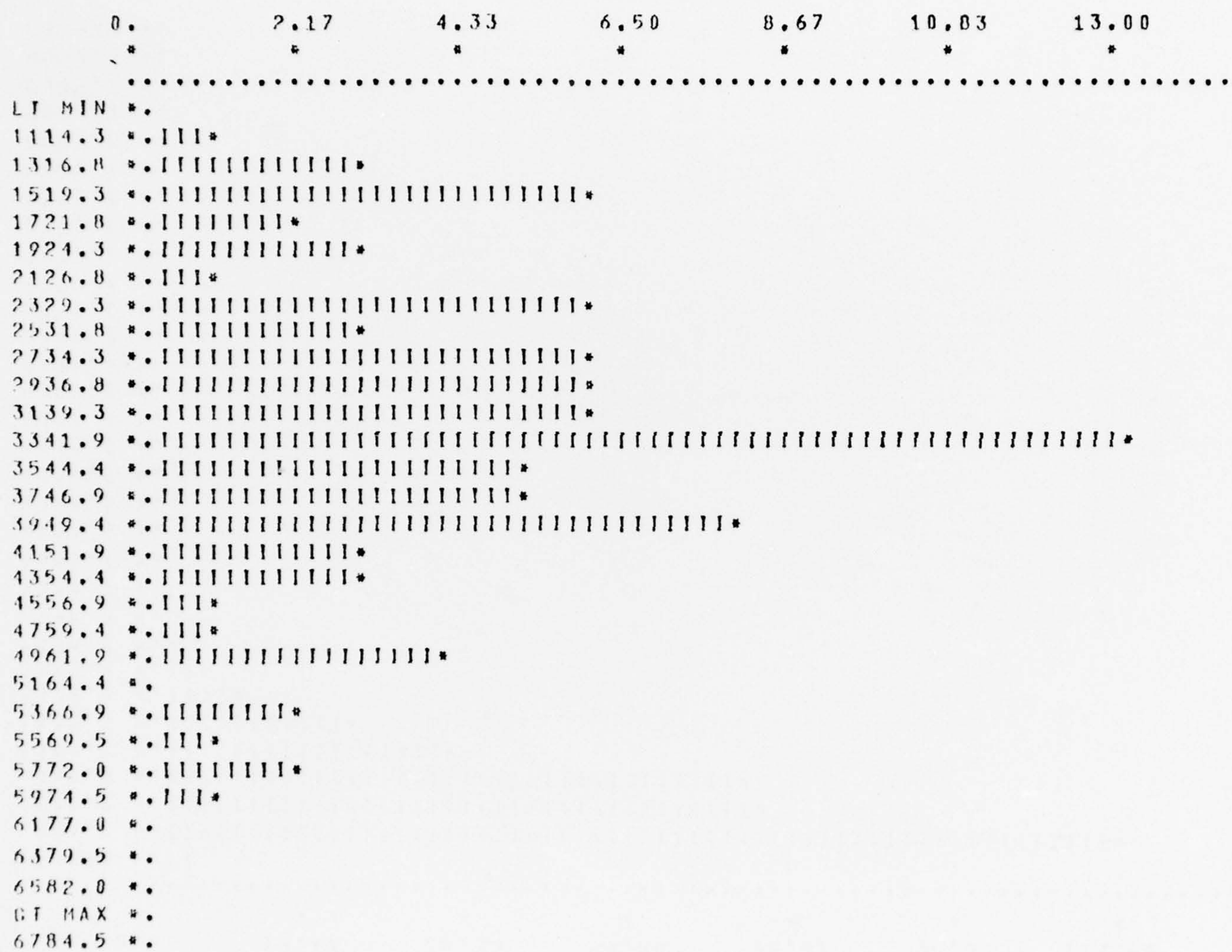
Histogram for LN-15 Cycle 3



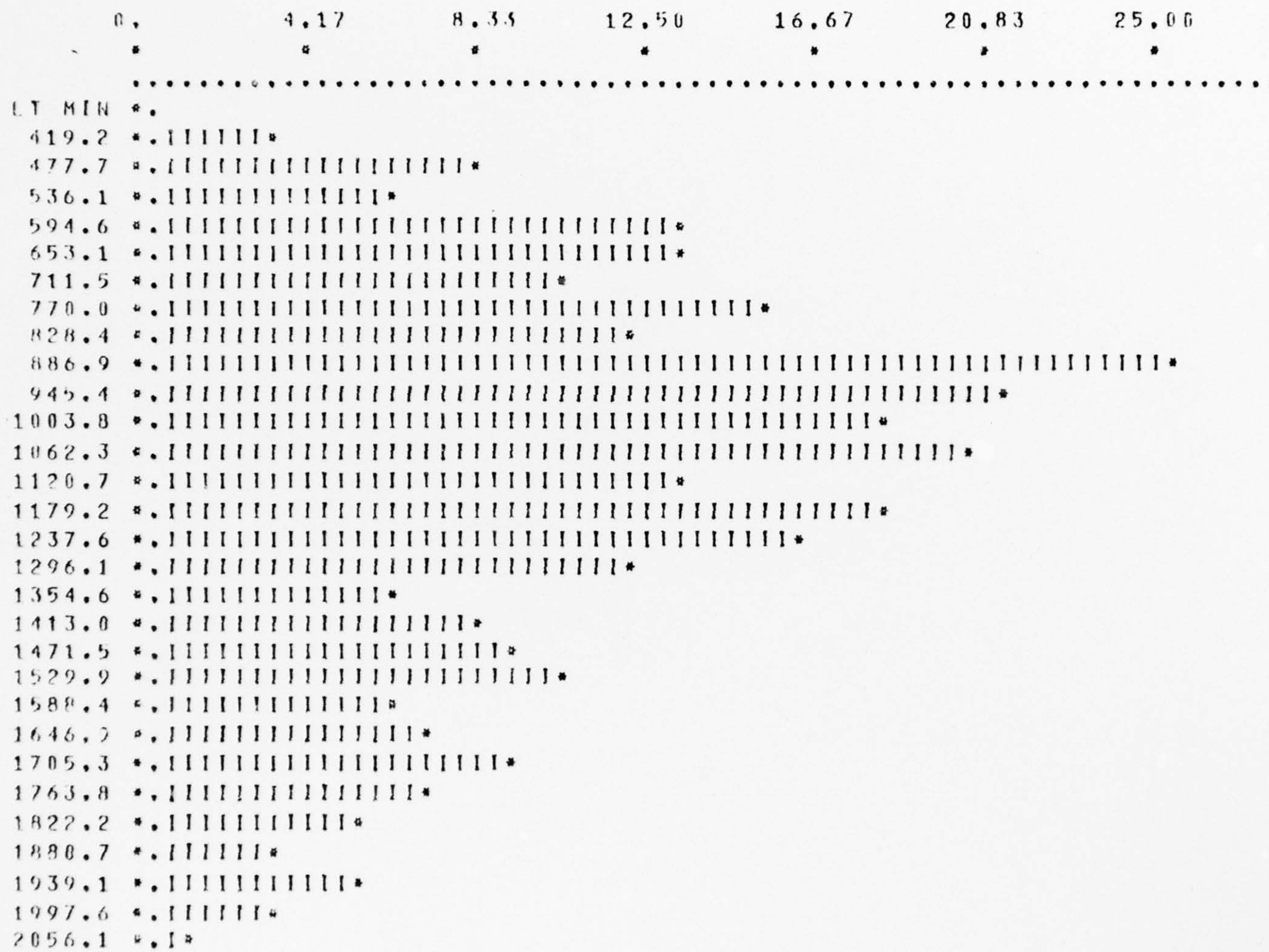
Histogram for LN-15 Cycle 4



Histogram of the Convoluted Data for KT-73



Histogram of the Convoluted Data for FLIP



Histogram of the Convoluted Data for LN-15

SELECTED BIBLIOGRAPHY

A. REFERENCES CITED

1. ARINC Research Corporation. Reliability Engineering. Edited by William H. Von Alven. Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1964.

2. Barlow, Richard R. Reliability and Product Assurance. Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1963.

3. Bazovsky, Igor. Reliability Theory and Practice. Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1961.

4. Budne, Thomas A. "Basic Philosophies in Reliability," Industrial Quality Control, September, 1961, pp. 87-93.

5. Clark, Charles T. and Lawrence L. Schkade. Statistical Analysis for Administrative Decisions. Cincinnati: South-Western Publishing Co., 1974.

6. Cox, D. R. Renewal Theory. New York: John Wiley and Sons, Inc., 1962.

7. Dellinger, David D. "Selected Reliability Test Plans," in W. Grant Ireson, ed., Reliability Handbook. New York: McGraw-Hill Book Company, 1966.

8. Dugas, Captain Louis A., USAF, and Captain David H. Hartman, USAF. "The Effects of Renewal Processes Upon Stochastic Reliability Models." Unpublished master's thesis, SLSR 16-76A, School of Systems and Logistics, Air Force Institute of Technology (AU), Wright-Patterson AFB, Ohio, June 1976.

9. Emory, C. William. Business Research Methods. Homewood, Illinois: Richard D. Irwin, Inc., 1976.

10. Enrick, Norbert Lloyd. Quality Control and Reliability. New York: The Industrial Press, 1966.

11. Erickson, Captain Richard F., USAF, and Captain Donald H. Hammond, USAF. "A Description of Expected Failure Rates of Newly Acquired Components Prior to Steady State." Unpublished master's thesis, SLSR 29-74A, School of Systems and Logistics, Air Force Institute of Technology (AU), Wright-Patterson AFB, Ohio, 1974.

12. Fibra, Isaac N. Probability and Statistical Inference for Scientists and Engineers. Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1973.

13. Gnedenko, B. V., K. Belyayev Yu, and A. D. Solovyev. Mathematical Methods of Reliability Theory. New York: Academic Press, Inc., 1969.

14. Hillier, Frederick S. and Gerald J. Lieberman. Operations Research. San Francisco: Holden-Day, Inc., 1974.

15. Hoel, Paul G. Introduction to Mathematical Statistics. New York: John Wiley & Sons, Inc., 1971.

16. Holden, James. Research Engineer, Oklahoma City Air Logistics Center, Tinker AFB, Oklahoma. Telephone interview. 22 April 1977.

17. Jorgenson, D. W., J. J. McCall, and R. Radner. Optimal Maintenance of Stochastically Failing Equipment. Santa Monica, California: The RAND Corporation, 1966.

18. Kennedy, Jerry. B-52, Technical Services, Oklahoma City Air Logistics Center, Tinker AFB, Oklahoma. Telephone interview. 22 April 1977.

19. Khintchine, A. Y. Mathematical Methods in the Theory of Queueing. London: Charles Griffen and Company, Limited, 1960.

20. Kirkpatrick, Elwood G. Introductory Statistics and Probability for Engineering, Science, and Technology. Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1974.

21. Landers, Richard R. Reliability and Product Assurance. Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1963.

22. Lloyd, David K., and Muron Lopow. Reliability: Management, Methods, and Mathematics. Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1962.

23. Moan, O. B. "Application of Mathematics and Statistics to Reliability and Life Studies," in W. Grant Ireson, ed., Reliability Handbook. New York: McGraw-Hill Book Company, 1966.

24. Pieruschka, Erich. Principles of Reliability. Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1963.

25. Roscoe, John T. Fundamental Research Statistics. New York: Holt, Rinehart and Winston, Inc., 1969.

26. Shooman, Martin L. Probabilistic Reliability: An Engineering Approach. New York: McGraw-Hill Book Company, 1968.

27. Siegel, Sidney. Nonparametric Statistics for the Behavioral Sciences. New York: McGraw-Hill Book Company, Inc., 1973.

28. Smith, Robert M. "Conceptual System Reliability Assessment," A Technical Report for the BDM Corporation, Albuquerque, New Mexico, 1976.

29. U.S. Air Force Logistics Command. Repair and Evaluation Data (RED) Management Program for Aircraft Inertial Systems. AFLC Manual 66-309. Wright-Patterson AFB, Ohio, 27 December 1976.

30. U.S. Air Force. Users Guide for SIMFIT. 1ACOMMG Pamphlet 100-20, 22 October 1975. 1st Aerospace Communications Group--Command, Offutt AFB, Nebraska, 1975.

31. U.S. Army Material Command. Quality Assurance Reliability Handbook. AMC Pamphlet No. 702-3. Washington, D.C., 28 October 1968.

32. U.S. Department of Defense. Definition of Effectiveness Terms for Reliability, Maintainability, Human Factors, and Safety. MIL-STD-721B, Washington, D.C.: Government Printing Office, 1966.

33. U.S. Department of Defense. Naval Air Specification Model A-7D/E Airplane Weapons System Reliability and Maintainability. AS-1652, 29 March 1968. Naval Air Systems Command, Washington, D.C., 1968.

34. U.S. Department of Defense. Reliability Prediction. MIL-STD-756, 15 May 1973. Defense Supply Agency, Washington, D.C.: Government Printing Office, 1973.

35. U.S. Department of Defense. Reliability Tests: Exponential Distribution. MIL-STD-781, 15 November 1967. Washington, D.C.: Government Printing Office, 1967.

36. U.S. Department of Defense. Requirements for Reliability Program. MIL-STD-785, June, 1965. Washington, D.C.: Government Printing Office, 1965.

37. U.S. Department of the Air Force. Reliability and Maintainability Programs for Systems, Subsystems, Equipment and Munitions. AFR 80-5. Washington: Government Printing Office, 1973.

B. RELATED SOURCES

Amstadter, Bertram L. Reliability Mathematics. New York: McGraw-Hill Book Company, 1971.

Arkin, Herbert and Raymond R. Colton. Tables for Statisticians. New York: Barnes & Noble, Inc., 1965.

Baldrige, Jeffrey W., USAF, and James F. Kenney, USN. "Effect of Maintenance Actions on the Reliability Performance of Four Line Replaceable Units in the A-7 Aircraft." Unpublished master's thesis. SLSR 18-76B, AFIT/SL, Wright-Patterson AFB, Ohio, 1976.

Brender, David M. "The Uncertainty of System Failure-Rate Prediction," Quality Control and Applied Statistics. January 1968, pp. 75-6.

Calabro, S. R. Reliability Principles and Practices. New York: McGraw-Hill Book Company, Inc., 1962.

Ewart, Park J., James S. Ford, and Chi-Yuan Lin. Probability for Statistical Decision Making. Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1974.

Hurd, Walter L., Jr. "Engineering Design and Development for Reliability Systems," in W. Grant Ireson, ed., Reliability Handbook. New York: McGraw-Hill Book Company, 1966.

Johnson, Norman L. and Samuel Kotz. Distributions in Statistics: Continuous Multivariate Distributions. New York: John Wiley and Sons, Inc., 1972.

Kao, John H. K. "Characteristic Life Pattern and Their Uses," in W. Grant Ireson, ed., Reliability Handbook. New York: McGraw-Hill Book Company, 1966.

Locks, Mitchell O. Reliability, Maintainability, and Availability Assessment. Rochelle Park, New Jersey: Hayden Book Company, Inc., 1973.

Pierskalla, William P., and John A. Voelker. "A Survey of Maintenance Models: The Control and Surveillance of Deteriorating Systems." Unpublished technical report for the Office of Naval Research, Northwestern University, 1975.

Pieruschka, Erich. Principles of Reliability. Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1963.

Ryerson, Clifford M. "Acceptance Testing," in W. Grant Ireson, ed., Reliability Handbook. New York: McGraw-Hill Book Company, 1966.

Schmid, Merle D. Basic Reliability Training Course, Volume I. East Hartford, Connecticut: Pratt & Whitney Aircraft, 1968.

Smith, Robert M. and Lee J. Bain. "Correlation Type Goodness-of-Fit Statistics with Censored Sampling," Communications In Statistics, 1976, pp. 119-132.

________ and ________. "An Exponential Power Life-Testing Distribution," Communications In Statistics, 1975, pp. 469-481.

U.S. Department of Defense. Reliability Prediction of Electronic Equipment. MIL-STD-217B, 20 September 1974. Defense Supply Agency, Washington, D.C.: Government Printing Office, 1974.

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